A LAGRANGIAN SUB-GRID MODEL FOR THE DISPERSION OF CLOUDS OF TRACERS

Federico Toschi[1], Irene M. Mazzitelli[2] & Alessandra S. Lanotte[3]
[1] *Department of Applied Physics and Department of Mathematics and Computer Science, Eindhoven University of Technology, Eindhoven 5600 MB, The Netherlands*
CNR-IAC, Via dei Taurini 19, 00185 Rome, Italy
[2] *Department of Physics and INFN, University of Rome "Tor Vergata"*
Via della Ricerca Scientifica 1, 00133 Rome, Italy
[3] *CNR ISAC and INFN, Sez. di Lecce, Str. Prov. Lecce Monteroni, 73100 Lecce, Italy*

Abstract Turbulence models are expected to satisfy the conflicting requirements of accuracy and computational efficiency. Here we discuss a new model that was recently developed in order to accurately and efficiently describe the dynamic of a clouds of tracer particles in Large Eddy Simulations of homogeneous and isotropic turbulent flows. The models incorporates the multi-scale nature of time and space turbulent velocity correlations that are essential in order to correctly reproduce the relative dispersion of multi-particle clouds. The model can be seen as an off-grid solver for the Eulerian velocity field at the positions of a given number of Lagrangian tracers that self-consistently move with it. Extensions to non homogeneous and isotropic turbulence as well as to the dynamics of particles will be discussed.

INTRODUCTION

The dispersion of particles by turbulent flows is a common phenomena in nature as well as in many industrial processes. Due to the wide range of scales involved in turbulent flows, it is standard practice to perform under-resolved numerical simulations that attempt at describing the dynamics of the largest scales while modeling the smaller ones. This general idea, dubbed Subgrid Scale Modeling, needs to be extended to the Lagrangian domain when one is interested in the study of the dynamics of particles.

Here we present a recently developed Lagrangian subgrid model for the dynamics of tracer particles that is capable of correctly modeling the multiscale (both space and time) nature of turbulence velocity fluctuactions [1]. This Lagrangian subgrid model has also the advantaged of being efficient, with a computational cost that grows with the number of particles and thus rather inexpensive for limited number of tracers.

THE SUBGRID MODEL

The model evolves the dynamics of a number of tracers, $\mathcal{N}$, that, in absence of (resolved) Eulerian velocity fluctuations, are self-consistently evolved by their modeled velocities. The velocity fluctuations are describes on scales $l_n = L_0/\lambda^n$ with $n = 0, \ldots, N_m - 1$, where L_0 is the integral scale and λ is a logarithmically scaling factor, larger than unity, that separates the resolved scales (we have arbitrarily taken it equal to $\lambda = 2^{1/4}$).

At each scale, l_n, we associate a typical velocity fluctuation, thus mimicking the typical fluctuation on an eddy of that size and given by $u_n = q_0\, k_n^{-1/3}$, where q_0 is a typical large scale velocity. The presence of the factor $k_n = 2\pi/l_n$ ensures that the Kolmogorov 1941 scaling is imposed. Associated to these scales and velocities one can associate the expected correlation times, $\tau_n = l_n/u_n$, that can be interpreted as the eddy-turnover times of the eddies.

Via a simple Ornstein-Uhlenbeck (OU) process it is easy to let the velocity fluctuations on the different scales fluctuate with a correlation time given by τ_n:

$$\zeta_n^{(i)}(t+dt) = \zeta_n^{(i)}(t)\, e^{-dt/\tau_n} + u_n \sqrt{1 - e^{-2dt/\tau_n}}\, g\,, \tag{1}$$

The total velocity of a single tracer, i, may be defined as the sum of all these multiscale and multitime correlations, over all scales n as follows:

$$\mathbf{v}^{(i)}(t) = \sum_{n=0}^{N_m-1} \boldsymbol{\zeta}_n^{(i)}(t)\,. \tag{2}$$

Such a simple procedure would however not ensure the fact that two close-by tracers must experience the same velocity. In order to introduce the space correlation we do not employ the $\boldsymbol{\zeta}_n^{(i)}(t)$ directly, in order to obtain the velocity of the particles, but we first compute the following averages, $\mathbf{v}_n^{(i)}(t)$, that are weighted by a function of the distance between tracers:

$$\mathbf{v}_n^{(i)}(t) = \sum_{j=1}^{\mathcal{N}} \boldsymbol{\zeta}_n^{(j)}(t) \cdot (1 - f_{l_n}(|\mathbf{x}_i - \mathbf{x}_j|))\,. \tag{3}$$

The function $f(r)$ is responsible for the weighting and, e.g., it can be taken to be linearly growing from 0 to 1, as the distance r grows from 0 to l_n, and equal to 1, if $r > l_n$. In this way the velocities of two nearby particles are correlated, scale-by-scale, only if at a distance smaller that the size of the eddy, r, otherwise these are uncorrelated.

RESULTS

In Figure 1 (left panels) we report the behaviour of the velocity at two scales for a couple of tracers, initially close, that separate in time. As it can be seen, while the original OU process produces totally uncorrelated signals, the modified velocities display strong correlations, and thus similar velocities, when tracers are close-by and are instead uncorrelated when tracers are distant from each other.

In Figure 1 (righ panel) we show that the modeled Lagrangian velocity is capable to correctly reproduce the relative separation as expected from Richardson t^3-law. It can further be shown that multiparticles distances evolve in close agreement with the results from Direct Numerical Simulations [1]. In addition the model is capable to correctly reproduce also one particle properties like e.g. absolute dispersion.

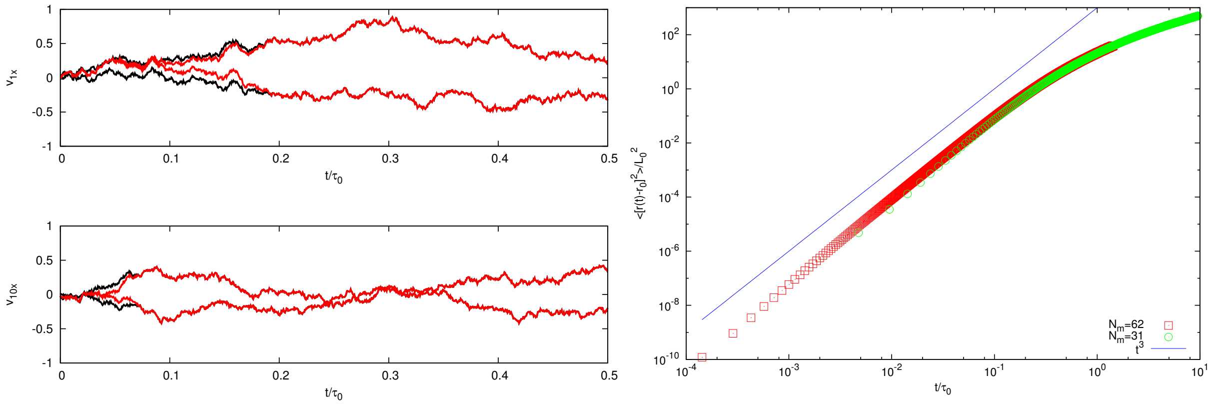

Figure 1: (Left panels) Velocities for a single pair of tracers that are initially at a distance r_0 smaller than the smallest eddy. Top-left panel: velocity fluctuation (only one component is shown) corresponding to the mode $n = 1$. Bottom-left panel: same but for the mode $n = 10$. The curves represent: the OU process is the solid black line while particle modulated velocities are the solid red lines. Righ panel: plot of tracers relative dispersion (in log-log scale) for the simulations with $N_m = 31$ (green circles) and $N_m = 62$ (red squares). The solid straight line indicates the slope for Richardson t^3 scaling regime that is expected in the inertial range.

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References

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