THE DECAY OF WALL BOUNDED MHD TURBULENCE AT LOW RM

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Abstract We have developed a new spectral method to simulate flows with very fine boundary layers present. We apply it to calculate the evolution of freely decaying MHD turbulence between isolating walls. By comparison them with results obtained in fully periodic domain we quantify the influence of the channel walls on the character of freely decaying MHD turbulence.

EQUATIONS

We consider a flow of liquid metal in channel configuration in a homogeneous magnetic field bounded by impermeable, electrically insulating walls located at $z = \pm 0.5L$ in a low Rm regime. Under these assumptions the set of governing equations can be expressed in dimensionless form:

$$\frac{\partial \mathbf{u}}{\partial t} + P(\mathbf{u} \cdot \nabla)\mathbf{u} = \Delta \mathbf{u} - \frac{1}{Ha^2}\Delta^{-1}\partial_{zz}\mathbf{u} \tag{1}$$

where $Ha = LB\sqrt{\sigma/\rho\nu}$, called the Hartman number, represents the ratio of Lorentz to viscous forces and P denotes orthogonal projection onto the subspace of solenoidal fields.

NUMERICAL METHOD

We express the solution of eq. (1) in terms of eigenvectors of operator that represents linear part of eq. (1). In [2] a set of solenoidal solutions to eigenproblem of operator has been derived. Because these modes are obtained from an operator that reflects the physics of this class of flow, the set of modes built this way is made of elements that reflect structures of the actual flows. Therefore they are natural candidates to use as a basis in a numerical spectral scheme. In particular, laminar and turbulent Hartmann boundary layers that develop along the channel walls appear as built-in features. Moreover it can be shown that to resolve the flow completely it is necessary to take into account all modes $\lambda < \lambda_{max}$ such as their number is equal to Re^2/Ha [2].

However in general the non linear terms cannot be expressed as an expansion in terms of the above modes, as they span only the divergence free subspace of all functions fulfilling the boundary conditions. Therefore our basis has to be supplemented with additional elements spreading the irrotational subspace as well. We obtain them by solving the eigenproblem with condition that velocity field is solenoidal replaced with condition $\nabla \times \mathbf{u} = 0$.

To calculate the non linear terms we use a pseudospectral approach and calculate them first in real space and next obtain their spectral expansion using collocation method (see [1] for more details).

DECAYING TURBULENCE

We used the potential of the new method to study the behaviour of the MHD turbulence in four sets of calculations with values of magnetic field corresponding to $Ha = 112$, $Ha = 224$, $Ha = 448$ and $Ha = 896$. We started all simulations from exactly the same initial conditions characterized by Reynolds number $Re = 178$ and evolved them up to $150\,\tau_J$, where $\tau_J = \rho/(\sigma B^2)$. The fig. 1 shows the evolution of the ratio of viscous (ϵ_ν) to Joule (ϵ_J) dissipation rates. In all cases the flow evolution can be split into two phases. In the first phase the energy decay is dominated by Joule dissipation. During this phase the flow becomes closer to a quasi 2D state due to the diffusion of the momentum along the magnetic field by the Lorentz force. In the second phase the energy is dissipated almost equally by viscous and Joule dissipations with the ratio of both dissipations reaching its maximum value and then slightly decreasing in time for all cases except $Ha = 896$. The maximum becomes less pronounced with increasing Ha, with the ratio of maximal value of ϵ_ν/ϵ_J to its asymptotic decreasing from 1.28 at $Ha = 112$ to 1.04 at $Ha = 448$. We presume that for $Ha = 896$ the maximum in evolution of ϵ_ν/ϵ_J is still present at time $\gtrsim 200\tau_J$ although even less prominent. The sharp initial increase of this ratio is due to the two-dimensionalisation of smaller and smaller structures. In the presence of Hartmann walls, quasi-two-dimensional structures generate little dissipation in the bulk. Most of their dissipation comes from the Hartmann layers, where viscous and Joule dissipation are locally of the same order. As time progresses, two-dimensionality is achieved in increasingly small structures and this explains that the ratio ϵ_ν/ϵ_J converges to a value of the order of unity. To characterize the influence of the walls we performed additional sets of calculations starting from exactly the same initial conditions as before but with the periodic boundary conditions imposed in all three directions. In the beginning the dissipation rate is again dominated by Joule dissipation. This phase is very similar to the one in cases with insulating walls. In the second phase the flow is again strongly two dimensional and the energy is dissipated mainly by viscosity. However the Joule

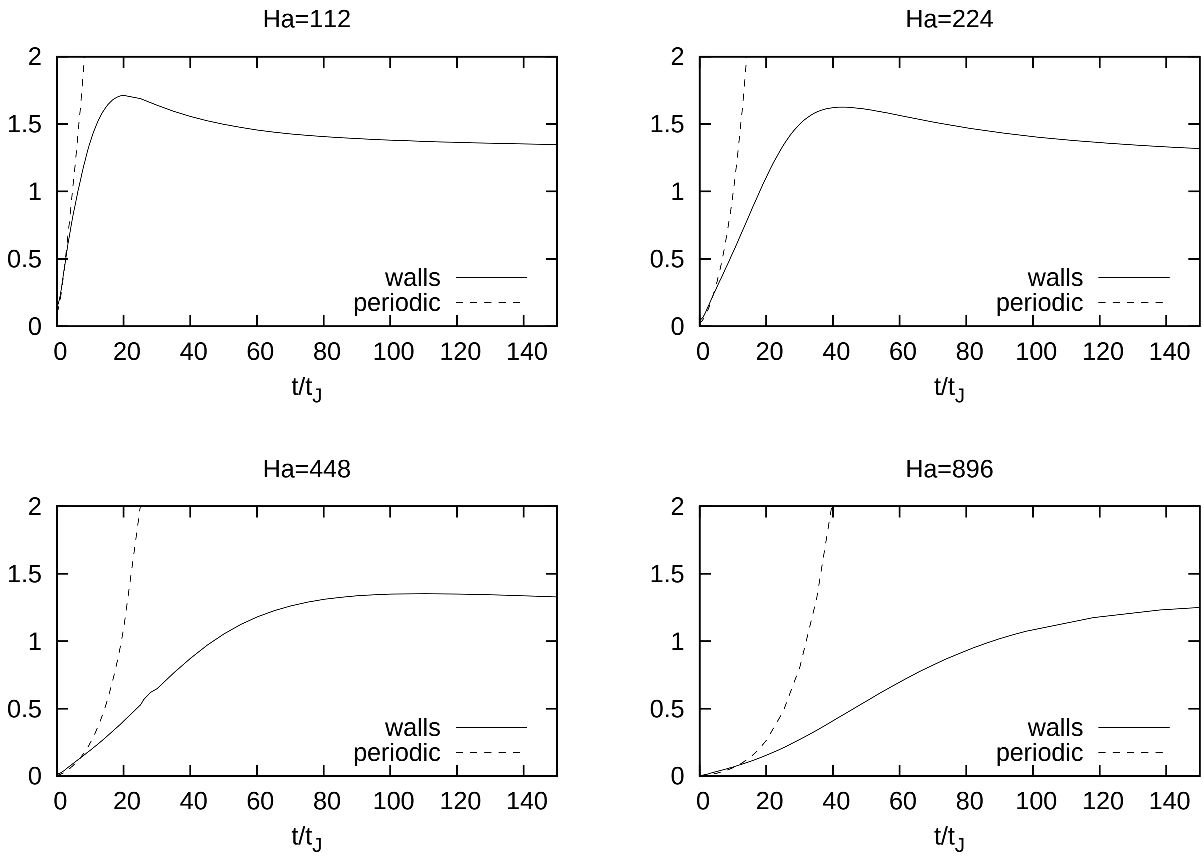

Figure 1. Evolution of ratio of viscous to Joule dissipation rates for $Ha = 112$, 224, 448 and 896. Solid lines denote the results for simulations bounded with walls, while dashed one denote results obtained using fully periodic domain.

dissipation decreases much faster with time with the ratio of viscous to Joule dissipations monotonically increasing with time. Consequently at the end of the simulation the Joule dissipation is negligible, which is in strong contrast to simulations with insulating walls. Our calculations confirm that the large scales are two-dimensionalized over $t_{2D} = \tau_J(L/l_\perp)$, where $l_\perp$ is an integral lengthscale in the direction perpendicular to the magnetic field, and was initially equal to 0.4 in our calculations. However our results also show that two dimensionalization of the whole spectrum takes place over a much longer timescale $t_H = Hat_{2D}$ and the evolution of energy and dissipation does not reflect a 2D behaviour until such later times.

References

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