TURBULENT FLOWS DRIVEN BY THE LIBRATION OF AN ELLIPSOIDAL CONTAINER

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Abstract We present a combination of laboratory experiments and numerical simulations modelling a geophysically relevant mechanical forcing: libration. Longitudinal libration corresponds to the periodic oscillation of a body's rotation and is, along with precessional and tidal forcings, a possible source of turbulence in the fluid interior of satellites and planets. In this study, we investigate the fluid motions inside a librating tri-axial ellipsoidal container filled with an incompressible fluid. The turbulent flow is driven by the elliptic instability which is a triadic resonance between two inertial modes and the base flow with elliptical streamlines. This is called the libration driven elliptical instability (LDEI)[2]. We characterize the transition to turbulence as triadic resonances develop while also investigating the properties of the turbulent flow that displays both intermittent or sustained regimes. The existence of such intense flows may play an important role in understanding the thermal and magnetic evolution of bodies subject to mechanical forcing, which is not considered in standard models of convectively-driven magnetic field generation.

MODEL AND METHODS

This work follows the recent experimental study by Grannan *et al.* [5]. We consider a tri-axial ellipsoid filled with a fluid of constant kinematic viscosity ν. The surface of the ellipsoid is defined by the cartesian equation $x^2/a^2+y^2/b^2+z^2/c^2=1$, and the ellipticity of the container is defined as $\beta=(a^2-b^2)/(a^2+b^2)$. The use of an ellipsoidal container is fundamental here, since the coupling between the solid boundary and the fluid is topographic and not only viscous as in the axisymmetric case. The ellipsoid is rotating around the vertical axis $\hat{\boldsymbol{z}}$ with a frequency Ω which is time dependent and given by $\Omega(t)=\Omega_0+\Delta\phi\,\omega_l\sin(\omega_l t)$ where $\Delta\phi$ is the libration amplitude and ω_l is the libration frequency. We work in the librating frame so that the boundaries of the ellipsoid are fixed. The equations of motion in this frame are

$$\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}+2\left(1+\epsilon\sin\left(ft\right)\right)\hat{\boldsymbol{z}}\times\boldsymbol{u}=-\nabla P+E\nabla^2\boldsymbol{u}-\epsilon f\cos(ft)\hat{\boldsymbol{z}}\times\boldsymbol{r} \tag{1}$$

$$\nabla\cdot\boldsymbol{u}=0\;, \tag{2}$$

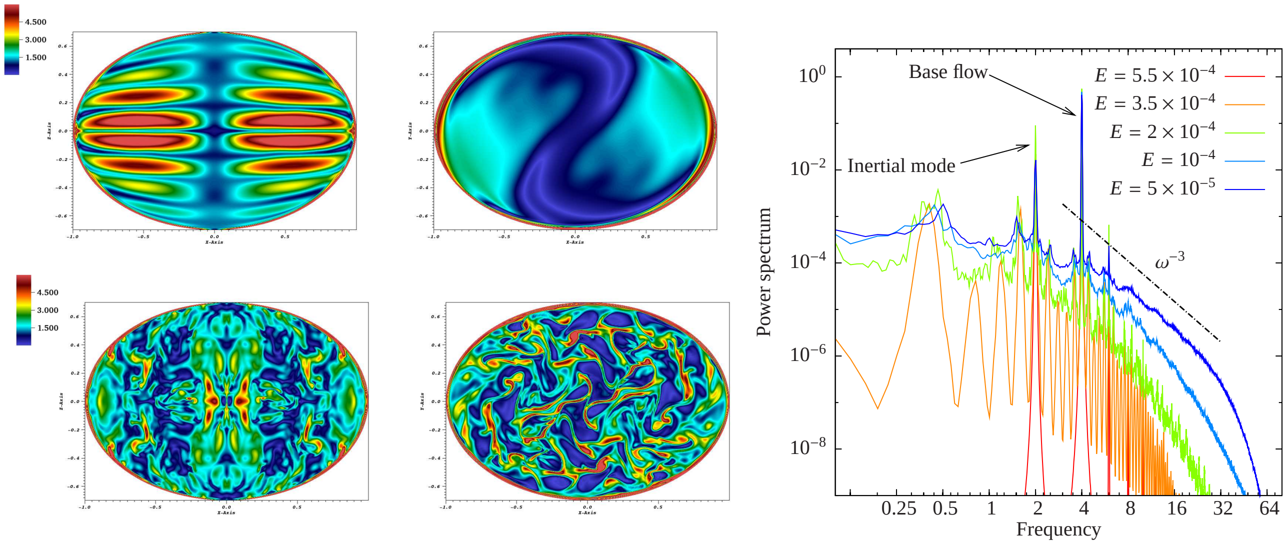

Figure 1. Left and middle: Enstrophy in meridional and equatorial slices for $E=5\times10^{-5}$ from numerical simulations. The top figures correspond to an arbitrary early time during the exponential phase of the instability where the inertial modes are clearly visible. The bottom figures correspond to the wave-breaking regime as the instability saturates. Right: Power spectrum averaged over 31 probes distributed within the ellipsoid. The spectral analysis is performed after the instability has saturated.

where we use the semi-major axis a as a length scale and Ω_0^{-1} as a time scale. The Ekman number is $E = \nu/(\Omega_0 a^2)$, $f = \omega_l/\Omega_0$ is the dimensionless libration frequency and $\epsilon = \Delta\phi\, f$ is the libration forcing parameter.

The experimental setup used in the present work is adapted from the same apparatus used previously in [6, 7, 5]. The enclosed fluid cavity is ellipsoidal with a long axis $a = 127$mm and short axes $b = c = 89$mm, leading to a fixed equatorial ellipticity of $\beta = 0.34$. A first motor rotates the turntable at a constant angular velocity $\Omega_0 = 30$rpm corresponding to $E = 2 \times 10^{-5}$ and the second, which is directly coupled to the acrylic ellipsoidal cavity, superimposes a sinusoidal oscillation whose parameter range is $[\Delta\phi, f] = [0.05 - 2.5, 0.5 - 9]$. More details about the experimental setup can be found in [5].

In addition to the experiments, we also use the spectral elements code Nek5000 to simulate the time evolution of the fluid initially at rest in the librating frame[4]. As in the experiment, the ellipsoid is characterized by an ellipticity $\beta = 0.34$, which correspond in our dimensionless units to a semi-major axis $a = 1$ and $b = c = 0.7$. The librating frequencies is varied in the range $f \in [1 - 4]$ and the librating amplitude is fixed to be $\epsilon = 0.8$. We vary the Ekman number from stable values down to $E = 5 \times 10^{-5}$, which only differs with the experimental value by a factor 2.5. The Nek5000 code has already been used in the context of tidally-forced rotating flows[3].

RESULTS

Here, we focus on the particular libration frequency $f = 4$. This frequency is ideal to study the transition to turbulence since the dominant triadic resonance must involve frequencies $f = 2$, which is the limit of existence of inertial modes. First, we recover the base and zonal flows driven by the libration forcing. The amplitude of this zonal flow does not scale with the Ekman number and scales as the square of the libration amplitude. Secondly, as the Ekman number decreases below a critical value of $E \approx 6\times10^{-4}$, the elliptical instability is observed. For $2\times10^{-4} < E < 6\times10^{-4}$, the exponential growth phase is followed by a quasi-stationary saturated state which is not turbulent. In this regime, we clearly identify the main resonance between the base flow at frequency $f = 4$ and inertial modes at $f = 2$ (see the right panel in Figure 1 for example). As the Ekman number decreases to smaller values, additional resonances are allowed, eventually leading to a fully turbulent regime for $E < 2 \times 10^{-4}$. The resulting turbulence is best characterised as rotating turbulence and the small-scale Rossby number is smaller than unity (see the left panel in Figure 1). The numerical results compare well with the experimental ones when looking at the generation of the zonal flow or low-frequency power spectra.

In addition to these preliminary results, we will also discuss the effect of reducing the ellipticity of the container as an effort to move towards the geophysically relevant regime. As β decreases, we switch from a sustained turbulent regime to an intermittent one, where bursts of turbulence are followed by viscously-dominated decay phases, similarly to what has been observed in tri-periodic shearing box simulations[1]. Note however that this is observed at a constant Ekman number $E = 10^{-4}$ so that by reducing β we also reduce the growth rate of the instability. It remains to be seen whether the elliptical instability saturates in an intermittent or quasi-steady manner in the geophysical regime at low-β *and* low-E.

References

[1] A. J. Barker and Y. Lithwick. Non-linear evolution of the tidal elliptical instability in gaseous planets and stars. *Monthly Notices of the Royal Astronomical Society*, **435**:3614–3626, 2013.

[2] D. Cébron, M. Le Bars, J. Noir, and J. M. Aurnou. Libration driven elliptical instability. *Physics of Fluids*, **24**(6):061703, 2012.

[3] B. Favier, A. J. Barker, C. Baruteau, and G. I. Ogilvie. Non-linear evolution of tidally forced inertial waves in rotating fluid bodies. *Monthly Notices of the Royal Astronomical Society*, **439**:845–860, 2014.

[4] P. F. Fischer, J. W. Lottes, and S. G. Kerkemeier. nek5000 Web page, 2008. http://nek5000.mcs.anl.gov.

[5] A. M. Grannan, M. Le Bars, D. Cébron, and J. M. Aurnou. Experimental study of global-scale turbulence in a librating ellipsoid. *Physics of Fluids*, **26**(12):126601, 2014.

[6] J. Noir, M. A. Calkins, M. Lasbleis, J. Cantwell, and J. M. Aurnou. Experimental study of libration-driven zonal flows in a straight cylinder. *Physics of the Earth and Planetary Interiors*, **182**:98–106, 2010.

[7] J. Noir, D. Cébron, M. Le Bars, A. Sauret, and J. M. Aurnou. Experimental study of libration-driven zonal flows in non-axisymmetric containers. *Physics of the Earth and Planetary Interiors*, **204**:1–10, 2012.