Multimodal instability and onset of the laminar-turbulent transition in a supersonic boundary layer

Dmitry Khotyanovsky[1,2] & Alexey Kudryavtsev[1,2]
[1]*Institute of Theoretical and Applied Mechanics, Siberian Branch of the Russian Academy of Sciences, Novosibirsk, Russia*
[2]*Novosibirsk State University, Novosibirsk, Russia*

Abstract 3D direct numerical simulation is employed for studying the flat-plate boundary layer instability evolving at high free-stream Mach number to study the non-linear interactions of the disturbances of different modes and the onset of the transition to turbulence.

INTRODUCTION

It is generally understood that the initial stages of the laminar-turbulent transition in high-speed boundary layers are governed by spatial growth of unstable small-amplitude disturbances. According to linear stability analysis and the available experimental data, at high Mach numbers the dominant boundary layer instabilities are those of the second mode, or Mack modes [1]. The oblique disturbances of the first mode are also unstable but have much lower growth rates. Linear and non-linear dynamics of the disturbed boundary layer thus involves complex interaction of the spectra of instability waves, competition of the modes, and may lead to entirely different transition scenarios depending on the conditions upstream. According to linear stability analysis with consideration of non-parallel effects, the spatial range of the second mode instability of a given frequency is rather small: as the boundary layer thickness (measured along the normal coordinate Y) increases downstream along X, initially unstable second mode disturbance of a given frequency quickly becomes stable and can no longer grow. We performed our own linear stability theory calculations using the formulation [2], and confirmed this. On the contrary, the oblique first-mode disturbance of a given frequency, though having lower growth rates than the second mode, can still be unstable farther downstream. The role of the mode competition in the laminar flow breakdown is not yet clear. Better understanding of the development of unstable disturbances of 1st and 2nd mode and their interactions is crucial for the design of efficient high-speed aircraft.

NUMERICAL TECHNIQUES

The objective of the present DNS study is to simulate interactions of the different modes of disturbances introduced at different frequencies, and find out what happens if they grow concurrently. In our DNS we solve numerically the 3D Navier–Stokes equations for compressible gas. The numerical computations are performed with the time-explicit Navier–Stokes numerical code based on a 5th order WENO scheme of Jiang & Shu [3]. Diffusion terms are computed on a compact stencil with central-biased differences. The code is accurate in time due to 4th order Runge–Kutta algorithm. The code is parallelized via domain decomposition and MPI. The simulations are performed with the assumption of the spatially evolving instability waves. Boundary conditions specify at inflow the self-similar laminar basic flow at a given Reynolds number Re with superimposed time-dependent fluctuations. Linear stability eigenfunctions of the unstable disturbances are used for the inflow forcing. Numerical simulation is typically performed with one most unstable two-dimensional wave of the second mode with a given frequency ω and two symmetrical 1st or 2nd mode instability waves propagating at angles χ and $-\chi$ to the basic flow in the transverse direction Z. The computational domain is long enough in X direction so that the disturbances have enough length to evolve. Sponge layer at the far end of the domain ensures damping of the disturbances near outflow. We use periodic conditions in transverse direction Z. Computations were run at flow Mach number M=6, Reynolds number based on Blasius thickness of the boundary layer at inflow boundary Re=1000, wall temperature ratio to the free-stream static temperature $T_w/T_e=7$. In our simulations we use grids condensed just above the plate and also resolving the critical layer, which is at 16-18 Blasius thicknesses above the plate. The entire mesh used herein was about 30 million grid cells. Computations were run at a distributed cluster using up to 64 CPU cores.

RESULTS

Here we present the results of the DNS of the development of the isolated unstable disturbances of the 2nd mode. Numerical results on the interaction of the two-dimensional 2nd mode disturbance with oblique 1st mode disturbances were presented at the ETC14. At flow Mach number M=6 the fundamental instability wave with frequency corresponding to the maximum growth rate of the linear theory is two-dimensional with the wave vector angle $\chi=0$. To provide the three-dimensional fluctuation field, a superposition of the 2D fundamental wave of the 2nd mode and two symmetrical oblique 2nd mode disturbances with angles χ and $-\chi$ was used as the inflow forcing. For boundary layer conditions at the inflow section, i.e. Re=1000, these were 35 and -35 degrees. Initial amplitude of the disturbances was 0.5%. Numerical results show that in accordance with linear theory the 2D fundamental wave is dominant at initial stages and rapidly grows in downstream direction. Farther downstream, because of the growth of the boundary layer thickness of the basic flow, the 2nd mode disturbance of a given frequency becomes stable. Hence, we also used a cascade approach for inflow forcing. In this approach, we superimpose on the basic flow the disturbance waves corresponding to the unstable disturbances at some cross-sections downstream, typically in the middle of the computational domain. In this case we also superimpose the oblique 2nd mode waves with $\chi=\pm 48°$. This cascade approach greatly prolongs the spatial region of the instability of a given frequency.

The results of the numerical simulations show that in all cases considered herein the initial stages of the instability development are mainly governed by the rapid growth of the fundamental instability of the 2nd mode. Oblique 2nd mode disturbances are also unstable but have much lower growth rates. Nevertheless, they are essential as they provide necessary 3D fluctuations. Farther downstream their effects become visible as three-dimensional deformations of the initially plane fluctuation field, which is evident in the results of our numerical simulations obtained with the cascade approach shown in Fig. 1. Farther downstream, as the planar fundamental wave stabilizes, the effects of the oblique 2nd mode disturbances become even more evident. These oblique instabilities efficiently pump energy in 3D fluctuations which leads to non-linear interactions and the onset of laminar-turbulent transition.

Comparison of these results with our previously reported results on the instability development in presence of the unstable oblique waves of the 1st mode suggests that the 1st mode is more efficient for triggering the laminar-turbulent transition.

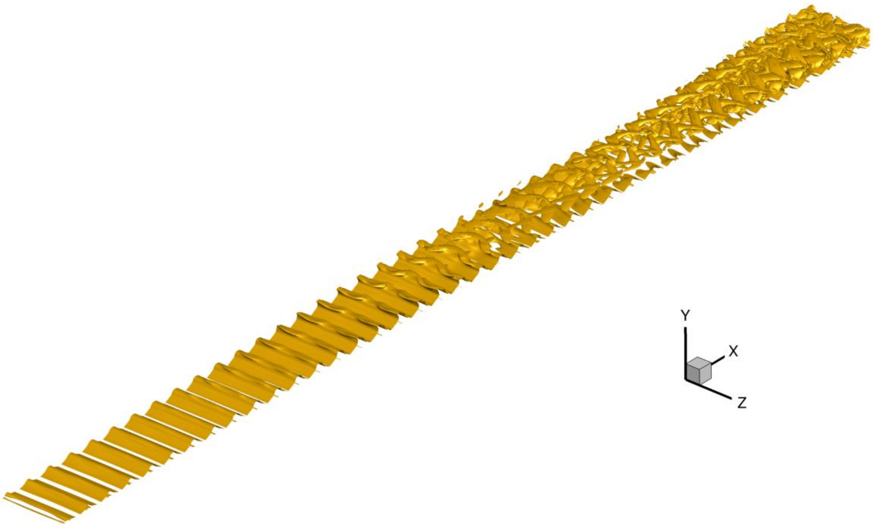

Figure 1. Second-mode instability and onset of the laminar-turbulent transition. Q criterion.

Acknowledgments

This study was supported by the RFBR (Grants 12-01-00840, 14-07-00065) and by the Government of the Russian Federation (Grant 14.Z50.31.0019). This support is gratefully acknowledged. The computations have been performed at the supercomputers of the Novosibirsk State University and the JSCC.

References

[1] L.M. Mack. Boundary layer stability theory. *Document 900-277, Rev. A. Pasadena, California, JPL*, 1969, 388 p.

[2] S. Özgen, S.A. Kırcalı. Linear stability analysis in compressible, flat-plate boundary-layers. *Theor. Comput. Fluid Dyn.* **22**: 1–20, 2008.

[3] G.-S. Jiang, C.-W. Shu. Efficient implementation of weighted ENO schemes. *J. Comput. Phys* **126**: 202–228, 1996.