INTERMITTENCY IN ELASTIC WAVE TURBULENCE

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Abstract We study numerically the long-time evolution of waves of a thin elastic plate for different energy input. In particular, we focus on the possible existence of intermittency, intended mainly as highly non-gaussian features. We show that deviations from the Kolmogorov- Zakharov scenario are present in high-order structure functions of the deplacement. This is more pronounced for higher-energy input even though the limit of small deformation so that modes of oscillations interact weakly is globally kept valid.

INTRODUCTION

In this work, we consider an oscillating thin elastic plate[3, 2, 6], which can be studied in the framework of wave-turbulence[7]. Plates are described through the dynamical version of Föppl-von Kàrmàn equations, which has been found to be an accurate model[3].
We perform numerical simulations of the full nonlinear system. In all the presented results the linear plate size is the only parameter of the numerics. We have used a pseudospectral scheme using FFT routines, with periodic boundary conditions: the linear part of the dynamics is calculated exactly in Fourier space. The nonlinear terms are first computed in real space and the integration in time is then performed in Fourier space using an Adams-Bashford scheme. It interpolates the nonlinear term as a polynomial function of time (of order one in the present calculations). As time evolves, the random waves oscillate with a disorganized behavior, as shown in Fig. 1.

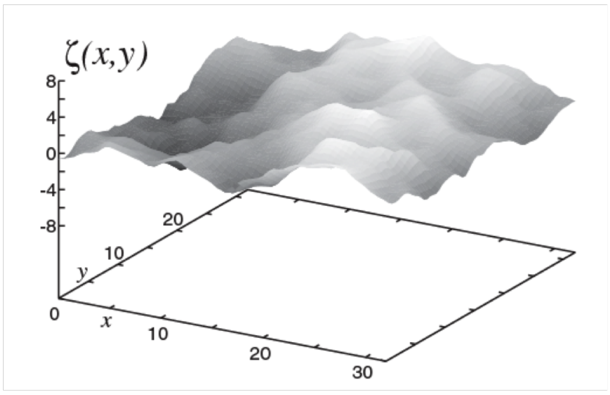

Figure 1. Zoom over a portion of the surface plate deflection $\zeta(x, y)$.

RESULTS

To analyse intermittency and anomalous scaling, that is lack of self-similarity, the relevant tool are the structure-functions [5]:

$$S_p(r) = \langle |\delta\zeta(\mathbf{x}, \mathbf{r})|^p \rangle,$$

where isotropy has been used and the increment is defined as $\delta\zeta(\mathbf{x}, \mathbf{r}) \equiv \zeta(\mathbf{x} + \mathbf{r}) - \zeta(\mathbf{x})$. It is worth emphasizing that statistics of deplacements and velocity are the same, since normal variables are a linear combination of both. Since the spectrum $E_\zeta \sim k^{-n}$, with $n \geq 3$ but a possible logarithmic correction, the cascade is not local and Wiener-Kinchin therorem does not apply, so that ζ field is expected to be smooth and thus $S_2(r) \sim r^2$. To cancel trivial scaling, higher-order difference should be used, notably 2nd-order $\delta\zeta^2(\mathbf{x}, \mathbf{r}) = \zeta(\mathbf{x} + \mathbf{r}) - \mathbf{2}\zeta(\mathbf{x}) + \zeta(\mathbf{x} - \mathbf{r})$. In this case, we have

$$S_2^2(r) = \langle (\delta\zeta^2(\mathbf{x}, \mathbf{r}))^2 \rangle \sim r^{(n-1)}. \tag{1}$$

In the case of a gaussian field, it is easy to show that for high-order structure functions $S_p^2(r) \sim r^{p(n-1)/2}$, which in the case of vibrating plates turns out to be $S_p^2(r) \sim r^p$.
In figure 2, we show the main results of our simulations obtained in statistically stationary averaging over 10^7 time steps to get much statistics. The exponent of second-order structure functions up to very high-order are shown. The exponent has

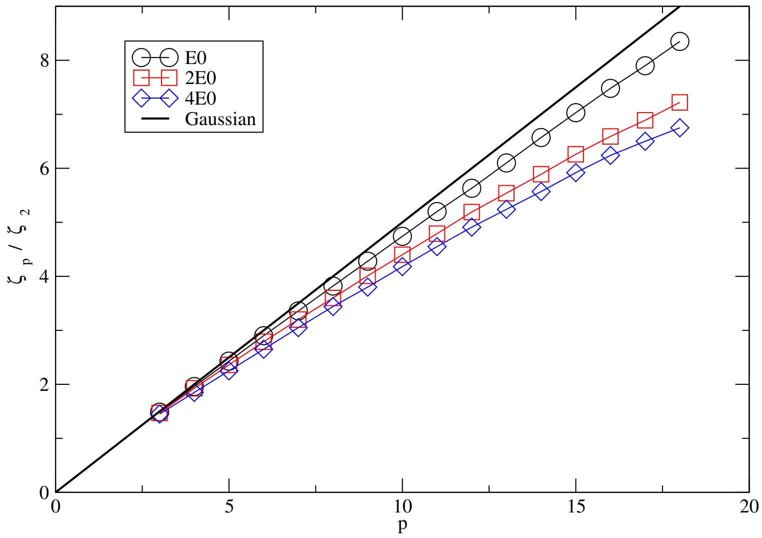

Figure 2. Structure function of the displacement for different p.

been computed through Extended-self-similarity (ESS) procedure[1], and in order to avoid imprecision due to the possible corrections to the spectrum, the exponent is shown normalized by the structure function with $p = 2$, that is correlation. Different energy inputs are used, the reference one is the weak-one used in previous calculations[3], which were shown to follow closely weak-turbulence theory. The gaussian prediction is also presented for comparison. Even though some questions can be raised about the very high-order structure functions ($p > 12$) because of the possible lack of statistics, it appears clear that:

1. Intermittency is always present, although it seems negligible in the weakest case for which gaussian prediction is reasonably accurate.

2. Intermittency grows with increasing energy input. In the most energetic case, large deviations appear as important as in strong turbulence[4].

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