ZERO-INERTIA INSTABILITIES IN RHEOPECTIC FLUIDS

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Abstract The emergence of fluid instabilities in the relevant limit of vanishing fluid inertia (i.e., arbitrarily close to zero Reynolds number) has been investigated for the well-known Kolmogorov flow. The time-lagged viscosity change from lower to higher values due to shear changes is the crucial ingredient for the instabilities to emerge. This behavior characterizes the so-called rheopectic fluids. The instability does not emerge in shear-thinning or -thickening fluids where viscosity adjustment to local shear occurs instantaneously. No instability arbitrarily close to zero Reynolds number is either observed in thixotropic fluids, even though the viscosity adjustment time to shear is finite like in rheopectic fluids. Numerical tools (through suitable eigenvalue problems from the linear stability analysis) and multiple-scale homogenization techniques are utilized to lead to our conclusions. Our findings may have important consequences in all situations where purely hydrodynamic fluid instabilities or mixing are inhibited due to negligible inertia, such as in microfluidics. To trigger mixing in these situations, suitable (not necessarily viscoelastic) non-Newtonian fluid solutions appear as a valid answer. Our results open interesting questions and challenges in the field of smart (fluid) materials.

I. INTRODUCTION

Control of mixing in fluid environments with very low Reynolds numbers is a need of paramount importance for many practical purposes. Applications range from biochemistry analysis in microfluidic devices, where mixing has to be rapid and efficient, to lab-on-a-chip applications, where mixing has to be reduced to avoid spurious effects as in microfluidic rheometer applications. For small Reynolds numbers, the resulting flow of a Newtonian fluid is typically laminar, and mixing occurs via diffusion. This mechanism is, however, extremely inefficient and slow. Fortunately, in many low-Reynolds number applications, including microfluidics, fluids are viscoelastic (a form of non-Newtonianity), a fact that has been recognized as an enormous advantage with respect to Newtonian fluids for the possibility of generating mixing via purely elastic instabilities. If properly triggered, these instabilities can originate the so-called elastic turbulence. Elastic turbulence is characterized by the algebraic decay of velocity power spectra over a wide range of scales and by its ability to generate more efficient mixing than in an ordered flow.

Elastic instabilities and elastic turbulence are characteristics of viscoelastic fluids: long polymer molecules added to a fluid make it elastic and capable of storing stresses that depend on the history of deformation, thereby providing the fluid a memory. The streamline curvature was thought to be a necessary ingredient to trigger the instability via a balance between normal stresses and streamline curvature. More recently, simple parallel flows clearly showed the emergence of purely elastic instabilities and turbulence even in the absence of curvature. Remarkably, the same class of viscoelastic parallel flows also displays other nontrivial viscoelastic properties including the well-known drag reduction by polymer additives. Our aim here is to show that the existence of purely non-Newtonian instabilities (a prelude to mixing in the nonlinear stage) occurring arbitrarily close to zero Reynolds number also exist for non-Newtonian fluids which are not viscoelastic.

The instabilities we have identified originate from a low-to-high-viscosity change. Such a change in the viscosity characterizes the so-called rheopectic fluids, which share with shear-thickening fluids the property that their apparent viscosities increase with strain. The crucial difference is that, for shear-thickening fluids, the response to strain is almost instantaneous while this is not so for rheopectic fluids. We found that this apparently innocent difference is the key point for purely non-Newtonian instabilities to emerge even for vanishing Reynolds number

II NON-NEWTONIAN MODEL

Let us start from the definition of the static version of our non-Newtonian fluid. Here, static is used to emphasize that the fluid response to stresses takes place instantaneously. The next step will be to incorporate a finite time response of the fluid that, as we will show, will be the crucial ingredient for network instabilities to emerge. A widely accepted model to describe the dependence on
shear rates of the apparent viscosity is the so-called Carreau-Bird model. According to this model, the expression for the viscosity is

$$\eta = \eta_0 (1 + 4\alpha^2 D_{ij} D_{ij})^{(n-1)/2} \qquad (1)$$

where η is the apparent viscosity and D_{ij} the strain rate tensor. α is a constant, and n (not necessarily integer) can be < 1 to model shear-thinning fluids or >1 to model shear-thickening ones) and is a measure of how far different fluid properties are from a purely Newtonian fluid.

Let us now proceed to include a fluid finite time response to shear. The idea is to introduce a kinetic equation for a scalar structure parameter λ in the same spirit of thixotropic modeling where such an equation is inspired by chemical kinetics. The resulting kinetic equation here simply follows from the requirement that Eq. (1) must be obtained in the limit of fast network response. By imposing this constraint one has

$$\eta = \eta_0 (1 + \lambda)^{(n-1)/2} \tag{2}$$

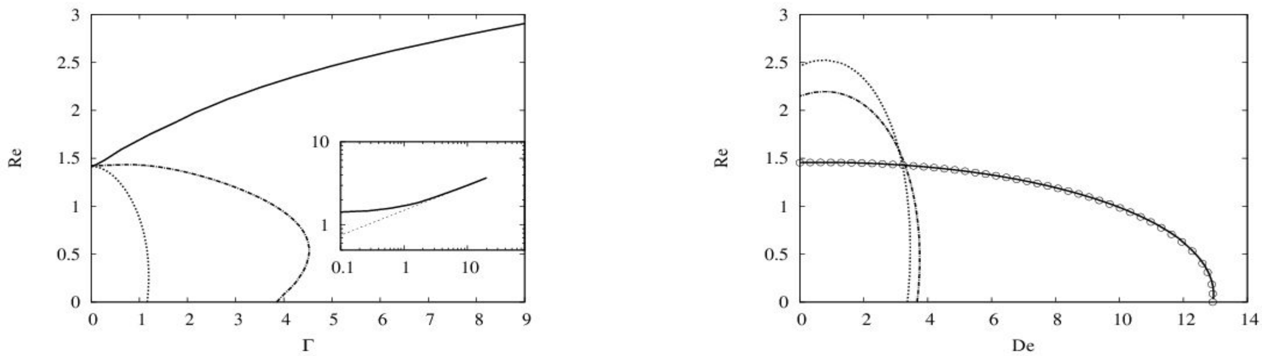
$$\frac{d\lambda}{dt} = -\frac{\lambda}{\tau} + 4\frac{\alpha^2}{\tau} D_{ij} D_{ij} \tag{3}$$

III. LINEAR STABILITY ANALYSIS

We will perform the linear stability analysis (by numerics and multiple-scale analysis) on the celebrated Kolmogorov flow $\boldsymbol{U} = (U(y),0,0)$ with $U(y) = V \cos(y/L)$, uniform pressure P (which can be set to zero) and a suitable forcing term in Navier-Stokes equation to force the base flow on this fixed point. We choose this flow because it will let us exploit multiple-scale methods along with numerical analysis. The following dimensionless free parameters enter into play: $Re = V L/\eta_0$, $De = \tau V/L$, and $\Gamma = aV/L$.

The results are represented in the following figures. In the left panel we show marginal curves in the parameter space Re-Γ for n = 1.3 and De = 0 (−), De = 3.5 (- · -) and De = 5 (· · ·). The inset is a log-log plot for the case De = 0 (−) showing a power-law behavior (- -). Below these curves we have stability; instabilities are found above the curves. De = 0 corresponds to the case of instantaneous response of the network. No network instabilities are observed in this case: the marginal curve increases with) as a power law (see inset) and it does not cross the Re = 0 axis. The situation drastically changes for finite De: depending on its value, marginal curves indeed cross the Re = 0 axis. This is the fingerprint of network instabilities. Although not shown, this property holds true for all n > 1.

In the right panel marginal curves are presented in the parameter space Re-De, again for n = 1.3, and Γ = 0.3 (−), Γ= 3 (- · -), and Γ= 5 (· · ·). In all cases, a critical De (as Re approaches 0) exists, whose value decreases for increasing Γ, associated with a transition to network instabilities. Once again, the key role of the finite-time response of the network to strain is crucial for network instabilities to emerge. Finally, the prediction from the multiple-scale expansion is shown with the symbol (o), and it fully corroborates the numerical results.



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