COMBINED EFFECTS OF PRESSURE GRADIENT AND BUOYANCY IN THE BOUNDARY LAYER OF A TURBULENT CONVECTION FLOW

Mikhail Ovsyannikov[1], Dmitry Krasnov[1], Mohammad S. Emran[1], Jörg Schumacher[1]
[1]*Institut für Thermo- und Fluiddynamik, Technische Universität Ilmenau, Ilmenau, Germany*

Abstract The new method for approximating the velocity and temperature of a flow within the boundary layers is developed by applying the combination of the Falkner-Skan approach and perturbation theory. The former enables to include non-zero pressure gradient along a heated horizontal plate where the flow is considered and the latter gives an opportunity to take into account buoyancy effects caused by the temperature difference between the hot plate and the flow above it. It is assumed that buoyancy effects are small. The mathematical model of the developed method includes four ordinary differential equations which are solved numerically. The approach is adapted to Rayleigh-Bénard convection considered in a cylindrical cell at aspect ratio one. The results obtained by the mathematical model and by direct numerical simulations of Rayleigh-Bénard convection are compared and are presented together with the conclusions made. The simulations were conducted for a closed cylindrical cell of aspect ratio one at the Rayleigh number $Ra = 3 \times 10^9$ and the Prandtl number $Pr = 0.7$ and $Pr = 7.0$.

Key words: boundary layer structure, Bénard convection, perturbation theory

INTRODUCTION

Turbulent Rayleigh-Bénard convection (RBC) occurs in a fluid confined between two isothermal horizontal plates heated below and cooled above. In addition, the fluid is restricted by vertical walls forming a closed cylindrical cell. A single large-scale circulation (LSC) is established in the cell of aspect ratio one. The thermal boundary layers are close to the horizontal plates and the viscous boundary layers form at all rigid walls. It was noted in [1] that the boundary layers at the horizontal plates of the cell can be transitional or even laminar for moderate Rayleigh numbers. This allows to apply Prandtl's equations describing a flow within laminar boundary layer to approximate the velocity and temperature of a flow within the boundary layers attached to the bottom horizontal plate. The set of Prandtl's equations can be reduced to an ordinary differential equation (ODE) and a similarity solution can be easily found. The reduction leads to the Falkner-Skan equation which is derived for non-zero pressure gradient if the distribution of the velocity above the boundary layer follows to a power or an exponential law. The former can be adapted to RBC in the cylindrical cell and is typical for a flow which runs along sides of a corner. The Falkner-Skan equation is used in [1] where the corner flow is observed in some area of the RBC cell but the buoyancy term is neglected. In the current work the buoyancy is additionally taken into account and its effect is to contribute to the longitudinal pressure gradient due to density variations which influence only the buoyancy force. Therefore the set of Prandtl's equations has two coupled equations and the reduction is not possible. In this case a similarity solution cannot be found as it was done in [1]. An idea to search for a series solution was developed in [2] where the buoyancy effects are considered as small but the longitudinal pressure gradient induced by a basic forced convection is zero. In the present work we discuss a laminar boundary layer model that contains effects of pressure gradient and buoyancy and compare the results with direct numerical simulation (DNS) data for turbulent Rayleigh-Bénard convection.

BOUNDARY LAYER MODEL

In the absence of the coupling between Prandtl's equations the reduction is done in two steps. The velocity components are expressed via the stream function ψ ($u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$) and the stream function ψ and temperature T are written via non-dimensional parameters $f = f(\xi)$ and $\theta = \theta(\xi)$, respectively, ξ is a new independent variable. These substitutions lead to the Falkner-Skan equation but here it is not possible due to the coupling. The idea suggested in [2] is to consider the buoyancy effects as small and to search for solutions in a form of series with some perturbative parameter ε: $f = \sum_{m=0}^{\infty} \varepsilon^m f_m$, $\theta = \sum_{m=0}^{\infty} \varepsilon^m \theta_m$. These series solutions give the perturbation of the forced convection flow due to the buoyancy. The zero-order terms f_0 and θ_0 are associated with the purely forced convection flow and are solutions of the equations of the Falkner-Skan case. Terms of higher order give the buoyancy effect. The series are truncated up to the first order and substituted into the set of Prandtl's equations. After substituting terms multiplied by ε^m with the same power m, $m = 0, 1$ are grouped. This results in the final set of four ODEs with the unknown functions f_0, f_1, θ_0, θ_1 depending on the variable ξ and their derivatives. Each ODE is solved numerically and independently in some fixed order, two ODEs are solved by the Adams-Bashforth method iteratively, others are solved by the simple Gaussian-elimination method. Thus the final set of equations forms the mathematical model and gives the series solutions which are shown for one case in figure 1 converted into the velocity along the axis x.

RESULTS

The approach developed in the current work was adapted to the RBC cell of a cylindrical form at the aspect ratio $\Gamma = 1$. The corner flow can be observed in the area where the LSC moves towards the bottom horizontal plate at some angle of attack, penetrates the boundary layers attached to this plate and runs along the plate. The angle of attack is determined via points at horizontal and vertical walls of the turbulent cell where the time-averaged wall shear stress τ_w is equal to zero. This can be seen as the green line in figure 2 where the mean flow which is aligned with the orientation of the large-scale circulation is also shown. The value of the angle is used to compute the power k which comes from the power law $U \sim x^k$ and is included in the equations of the mathematical model. After determining the coefficient k the series solutions of f and θ are computed by solving the model equations. Then the series solutions are used for computing the velocity and temperature of the flow within the boundary layers. The angle of atack varies depending on time used for averaging and control parameters used in performing DNS (Ra, Pr, Γ). Thus parameters involved in the mathematical model are taken from DNS and results obtained from both sources are compared then. We use DNS at Rayleigh number $Ra = 3 \times 10^9$ and Prandtl numbers $Pr = 0.7$ and $Pr = 7.0$. Before comparing results units used in the mathematical model must be converted into ones applied in DNS. There are several ways of converting units which are based on the equality of values of the thickness of corresponding boundary layers, on the coincidence of the first extremums of profiles and on the equality of the tangent of profiles at the wall. Our analysis, which can be considered as the final stage of what can be achieved within the framework of laminar boundary layer theory, shows that the additional effect of the buoyancy remains small compared to the one by the pressure gradient (see again figure 1 where model results and DNS data are compared). This holds also for $Pr > 1$. In all cases an improvement compared to the Blasius case was found.

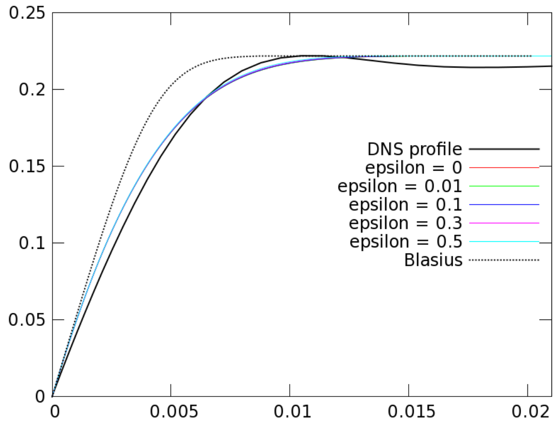

Figure 1. The comparison of the results of the velocity component along the axis x obtained by the mathematical model and DNS at $Pr = 0.7$.

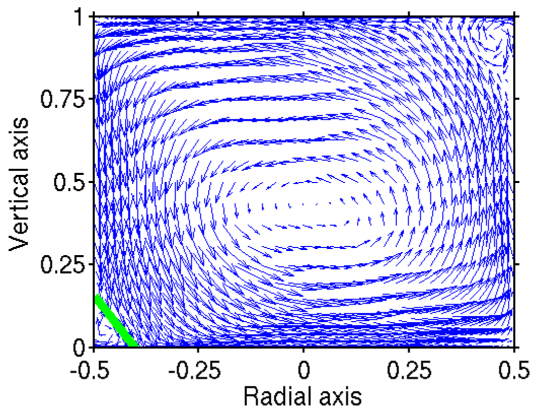

Figure 2. The vector field of the velocity components (u_r, u_z) averaged over time and taken at a vertical plane. The plane is aligned with the instantaneous orientation of the LSC, $Pr = 0.7$.

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