MAGNETOHYDRODYNAMIC TURBULENCE IN A HARTMANN DUCT FLOW AT FINITE MAGNETIC REYNOLDS NUMBER

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Abstract The dynamics of turbulent flow at finite magnetic Reynolds numbers can be very complex due to the coupled nature of the evolution equations for the flow and magnetic fields. In this regime, the Hartmann flow in a straight rectangular duct with streamwise periodicity is studied with the help of direct numerical simulations (DNS) and the effect of magnetic Reynolds number on turbulent statistics is quantified by comparing the results with the numerical results obtained using the quasistatic approximation.

INTRODUCTION

Magnetohydrodynamics (MHD) deals with the interaction of electrically conducting flows with a magnetic field. Such flows are commonly found in nature, for e.g. in stars and galaxies, in sunspots and solar flares, and in the maintanence of earth's magnetic field by the dynamo action. Significant industrial applications include control of metallurgical melt flows in the continuous casting of steel and aluminium, plasma confinement in nuclear fusion applications and material processing. Turbulence in the presence of a magnetic field encumbers several unique features that are not found in pure hydrodynamic fluid turbulence. Some of these features include strong anisotropy (preferred direction parallel to the magnetic field), an additional energy dissipation mechanism (joule heating) that occurs at all scales, quasi-two-dimensional turbulence and in case of wall bounded flows, the presence of distinctive boundary layers namely the Hartmann and Shercliff layers (see for e.g. [1]).

In a broad sense, MHD flows can be classified using a non-dimensional parameter called the magnetic Reynolds number, that is defined as

$$R_m = \frac{UL}{\lambda}, \tag{1}$$

where U and L are the charateristic velocity and length scales in the flow and λ is the magnetic diffusivity of the fluid given by $\lambda = (\mu_0 \sigma)^{-1}$, μ_0 and σ being the magnetic permeability of free space and the electrical conductivity of the fluid respectively. It is a measure of the relative magnitude of advection to diffusion of magnetic field in the flow. Typical industrial flows are of low to moderate R_m whereas the geodynamo and astrophysical flows involve high R_m ($R_m \gg 1$).

MHD TURBULENCE IN DUCT FLOWS

Direct numerical simulation of MHD turbulence in shear flows have mostly been performed using the so called 'Quasistatic' approximation which is valid when $R_m \ll 1$ (e.g.[2, 3]). Over the last decade, there has been a significant leap in our understanding of shear flow MHD turbulence at low R_m. As an example, Fig. 1 shows the preferentially elongated turbulent structures residual in the Shercliff layers of a Hartmann duct flow at a very high Reynolds number, $Re = 10^5$ and a moderate Hartmann number, $Ha = 350$ [3]. It can be seen that turbulence is full suppressed in the Hartmann layers.

The focus of the present work falls in the category of $R_m \sim 1$, wherein interesting phenomena in turbulence characteristics can be expected due to the coupled evolution of the flow and the magnetic fields. An important technological application of a flow in such a regime is in the transient Lorentz Force Velocimetry (LFV), which refers to the reconstruction of velocity fields from the measurement of Lorentz forces on permanent magnets placed without contact and in the vicinity of the flow [4].

In this work, we consider the case of flow of an electrically conducting fluid (e.g. a liquid metal) in a straight rectangular duct, driven by a mean pressure gradient in the presence of a uniform magnetic field perpendicular to two walls (Hartmann flow). In addition, the magnetic Reynolds number is assumed to be non-negligible and the flow exterior is considered to be electrically insulating. The coupled evolution of the flow and magnetic fields inside the duct as governed by the Navier-Stokes and the induction equation for the velocity and magnetic fields respectively is numerically solved using a full MHD direct numerical simulations (DNS) code that has been developed incorporating the integral magnetic boundary conditions to ensure proper matching of the interior secondary magnetic field with that of the exterior. Details of the numerical procedure will be presented. Furthermore, turbulence statistics obtained from the simulations are compared to the existing results from the quasistatic simulations highlighting the differences.

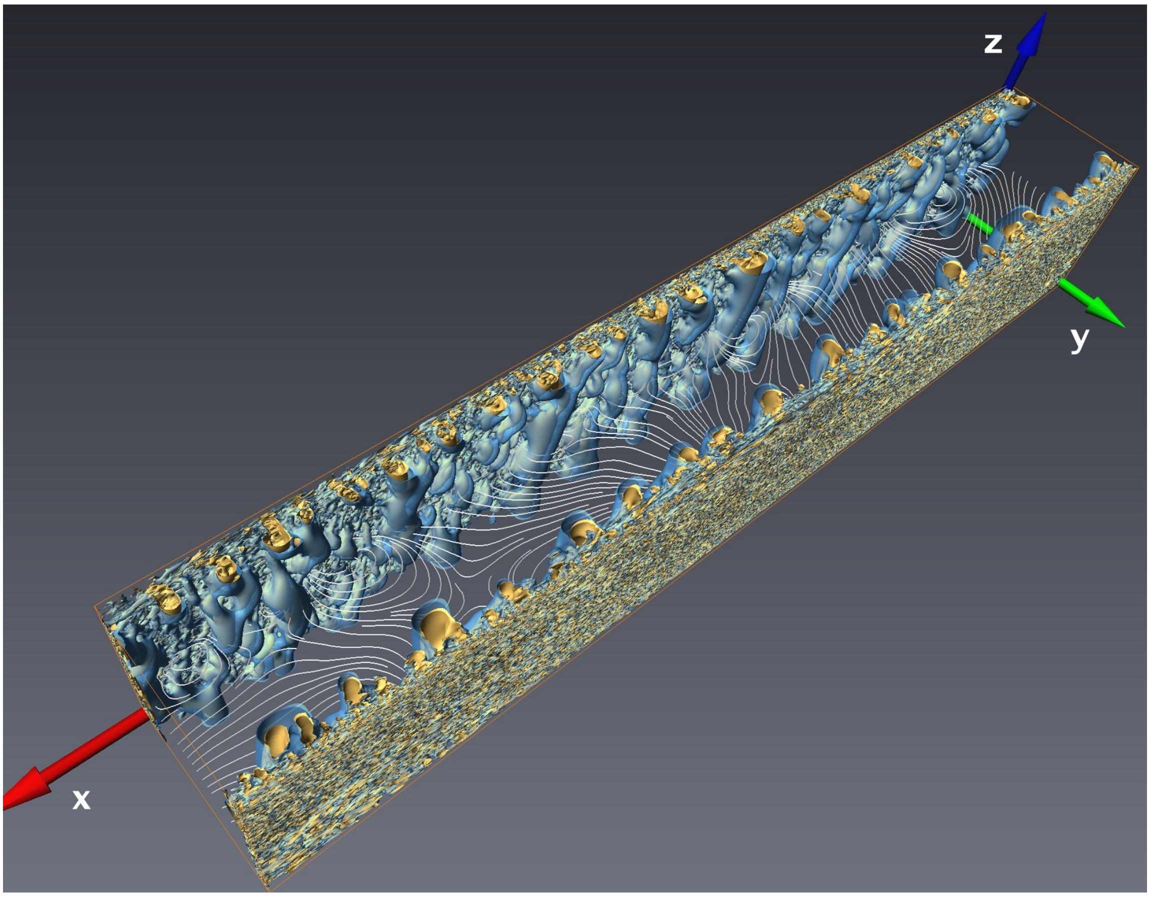

Figure 1. Instantaneous distribution of coherent vortices shown as isosurfaces of the second eigen value λ_2 of tensor $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$, along with streamlines of velocity fluctuations in the mid-plane at $Re = 10^5$ and $Ha = 350$.

References

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