LINEAR STABILITY OF A LIQUID FLOW THROUGH A POROELASTIC CHANNEL

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Abstract A liquid flow through a channel is studied based on the Orr-Sommerfeld eigenvalue problem, where the lower wall of the channel is occupied by the saturated poroelastic medium. The linear stability analysis is investigated in detail for arbitrary value of the wavenumber. The eigenvalues are computed numerically by using the Chebyshev spectral collocation method. The effect of physical parameters, for instance, permeability, elasticity as well as their combined effect on the unstable modes are examined.

INTRODUCTION

The flow through a poroelastic medium has recently triggered a considerable interest because of its several applications in biological problems, such as flow through a blood vessel where the blood vessel can be considered as a poroelastic medium. The transport of blood through a vessel plays an important role in maintaining metabolism of the human body and blood balance of the surrounding tissues [5]. Flow over poroelastic medium is also of interest for biomimetic applications aiming to reduce turbulent drag. Further, poroelastic materials have been employed in wide range of technological processes, for instance, fluidic pump driven by elastic filament [2] and nanorod arrays used in DNA analysis and separation [1]. In general, poroelastic materials are biphasic, made of a solid skeleton along with interconnected pores through which fluid passes and causes deformation to the solid skeleton due to elasticity. In order to investigate these types of flow problem in detail, we consider a model of pressure driven channel flow where the lower wall of the channel is occupied by the poroelastic medium. The sketch of the flow configuration can be found in figure 1. We implement two-domain ap-

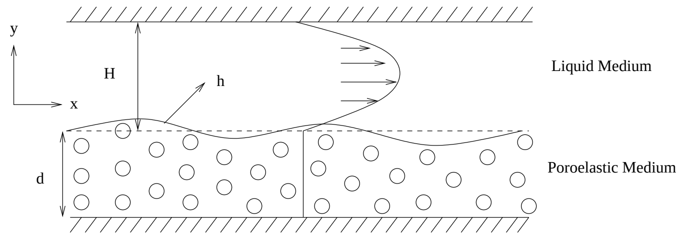

Figure 1. Sketch of a liquid flow through a poroelastic channel.

proach to deal with this problem, i.e., liquid and poroelastic media are treated as distinct medium separated by an interface and consequently, interface boundary conditions are required to complete the system. In the liquid medium, the flow is governed by the Navier-Stokes equations. As, the flow in the poroelastic medium is sufficiently slow in comparison with the flow in the liquid medium, we focus only on the leading order macroscopic equations, i.e. $\mathcal{O}(\varepsilon^0)$ equations, where $\varepsilon = l/L \ll 1$, L is the characteristic length scale of the poroelastic domain in macroscopic sense and l is the characteristic length scale of pore in microscopic sense. These macroscopic equations are valid in the entire poroelastic medium and are obtained by using the method of homogenization theory [3].

NUMERICAL EXPERIMENT

In order to study the linear stability analysis of the coupled boundary problem for arbitrary wavenumber, we use the procedure as proposed by [4]. First of all, we recast the Orr-Sommerfeld boundary value problem into a 4×4 generalized matrix eigenvalue problem

$$\mathcal{A}\mathbf{X} = \omega\mathcal{B}\mathbf{X}, \tag{1}$$

where $\omega = ck$ is the eigenvalue, $\mathcal{A}$ and $\mathcal{B}$ are generalized matrices and $\mathbf{X} = [\phi, \tilde{\phi}, \tilde{\chi}_1, \tilde{\chi}_2]^T$ is the corresponding eigenvector. The above eigenvalue problem (1) is resolved based on the Chebyshev spectral collocation method [4]. In this method, each perturbation amplitude function in the column matrix $\mathbf{X}$ is expanded as a series of Chebyshev polynomials

$$\phi = \sum_{i=0}^{N} \phi_i T_i(y), \quad \tilde{\phi} = \sum_{i=0}^{N} \tilde{\phi}_i T_i(y) \quad \tilde{\chi}_1 = \sum_{i=0}^{N} \tilde{\chi}_{1i} T_i(y), \quad \tilde{\chi}_2 = \sum_{i=0}^{N} \tilde{\chi}_{2i} T_i(y). \tag{2}$$

where ϕ_i, $\tilde{\phi}_i$, $\tilde{\chi}_{1i}$ and $\tilde{\chi}_{2i}$ are unknown coefficients to be determined. As, the Chebyshev polynomials $T_i(y)$ are defined over the domain $[-1, 1]$, thereby, it is necessary to transform the liquid-layer domain $[0, 1]$ to $[-1, 1]$ and consequently, we take a mapping $y = (x + 1)/2$, where $x \in [-1, 1]$. Similarly, for poroelastic layer $[-\delta, 0]$, we take a mapping

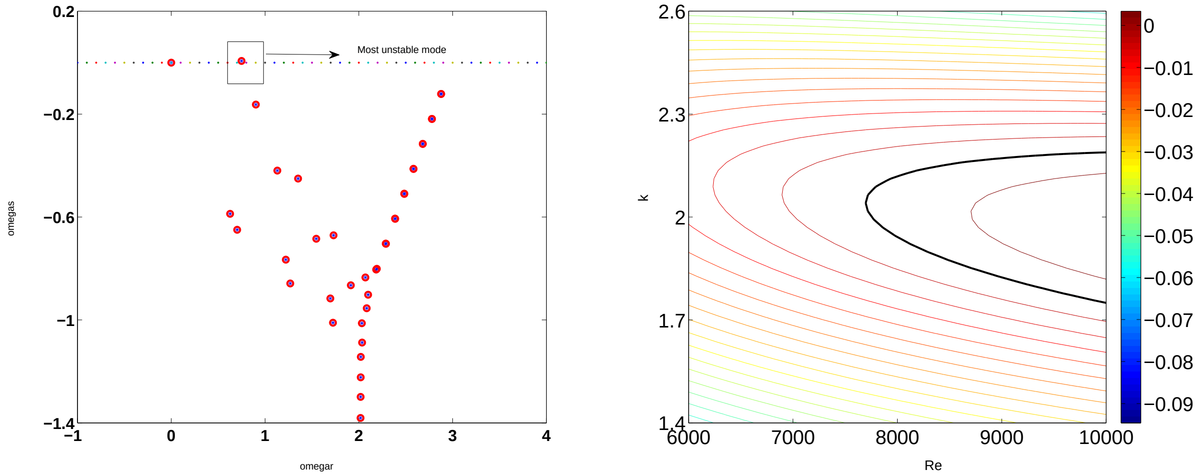

Figure 2. (a) The spectrum of the eigenvalue problem (1) in the limit of high elasticity and low permeability, when $k = 2$, $Re = 10000$. (b) The growth rate of the dominant unstable mode in the Reynolds number versus wavenumber plane.

$y = \delta(x-1)/2$, where $x \in [-1, 1]$. As a result, the derivatives are transformed to $D = 2D$, $D^2 = 4D^2$, $\cdots$ for liquid layer and $D = (2/\delta)D$, $D^2 = (4/\delta^2)D^2$, $\cdots$ for poroelastic layer. Here we keep at least hundred Chebyshev polynomials ($N \geq 100$) corresponding to each amplitude function in order to maintain good accuracy in the numerical results. Further, the present numerical code is tested with the known result for plane Poiseuille flow [4] by assuming very low permeability and very high elasticity. Basically, in this limit, the poroelastic layer behaves as a rigid solid layer and the entire system automatic transforms into a pressure driven Poiseuille flow between impermeable walls. Figure 2 represents the result of limiting case. The main purpose of this study is to decipher a phase boundary in the parameter space where the individual effect of permeability and elasticity on the most unstable modes are dominant. Further, the results are interpreted in terms of the ratio of the two characteristic time scales of the problems: the poroelastic time scale and the hydrodynamic time scale.

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