INTER-VORTEX SPACING IN SUPERFLUID TURBULENCE : TEMPERATURE AND REYNOLDS NUMBER DEPENDENCES

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Abstract The typical spacing between superfluid vortices in an isothermal turbulent tangle is proportional to the integral scale H rescaled by the quantum Reynolds number $Re_\kappa = H \cdot V \cdot \kappa^{-1}$ to the power of 3/4, where κ is the quantum of circulation around of single vortex [1]. This empirical relation can be seen as the quantum-turbulence version of the corresponding well-know equation giving Kolmogorov dissipative scale in classical turbulence. In 2014, we studied the temperature dependence of the numerical factor $\delta/H \times Re_\kappa^{3/4}$ in ^{4}He by joint numerical and experimental analysis of steady state turbulence over a wide temperature interval (1.2 - 2.16 K) [2]. Agreement between the two analyses was found good except at the very ends of this temperature interval. We will discuss this issue by presenting additional experimental data obtained by post-processing of superfluid experiments published between 1975 and 1998 [3, 4, 5].

INTRODUCTION

In classical turbulence, the small dissipative Kolmogorov length scale η relates to the integral scale H through the Reynolds number Re following :

$$\frac{\eta}{H} \simeq Re^{-3/4}.$$

Superfluid turbulence can be seen as a tangle of superfluid vortices with a quantized circulation of velocity κ around each vortex. A natural small length scale of the problem is the typical inter-vortex spacing δ. When a superfluid is mechanically stirred at the large scale, turbulence is produced and one can define an integral scale H and a root-mean-square velocity V in a fashion similar to classical flow. The constant κ, having the dimension of a kinematic viscosity, allows an introduction of a "superfluid" or "quantum" Reynolds number Re_κ sometimes defined as:

$$Re_\kappa = \frac{H \times V}{\kappa}.$$

In 2011, it was noticed that the dimensionless quantum scale δ/H is also proportional to $Re_\kappa^{-3/4}$, with a numerical prefactor of order one [1]. Recently [2], we noticed that this prefactor can be written as the $1/4$-power of a temperature dependent Schmidt number parameter Sc

$$\frac{\delta}{H} = Sc^{1/4} \times Re_\kappa^{-3/4}.$$

A joint numerical and experimental analysis allowed us to determine the temperature dependence of Sc in steady state flow. The quantity is plotted in the figure 1. The numerics were performed using the HVBK model; although superfluid vortices are smoothed out in this continuous model, an equivalent vortex line density can be estimated from the superfluid vorticity. Experiments were performed using second sound attenuation technique, sensing the vortex line density in a channel flow generated mechanically by squeezing a bellows (with or without a grid obstruction upstream from the detection region). As can be seen in figure 1, the temperature dependence is found more pronounced in the numerics than in the steady-state experiments, in particular at the upper and lower ends of the temperature range.

NEW ANALYSIS

Many experimental measurements of δ in superfluid flows have been performed either in decay experiments or in steady state thermally-driven flows, but these measurements do not allow a straightforward test of the above equation, nor a straightforward determination of Sc. Indeed, thermally-driven flows are different in nature from the mechanically-driven flows of interest here, due to different forcing of the individual fluid components of He-II. Regarding decay experiments, they provide an effective viscosity parameter ν_{eff} which can be related to the Schmidt number through $Sc = \nu_{eff}/\kappa$ (as

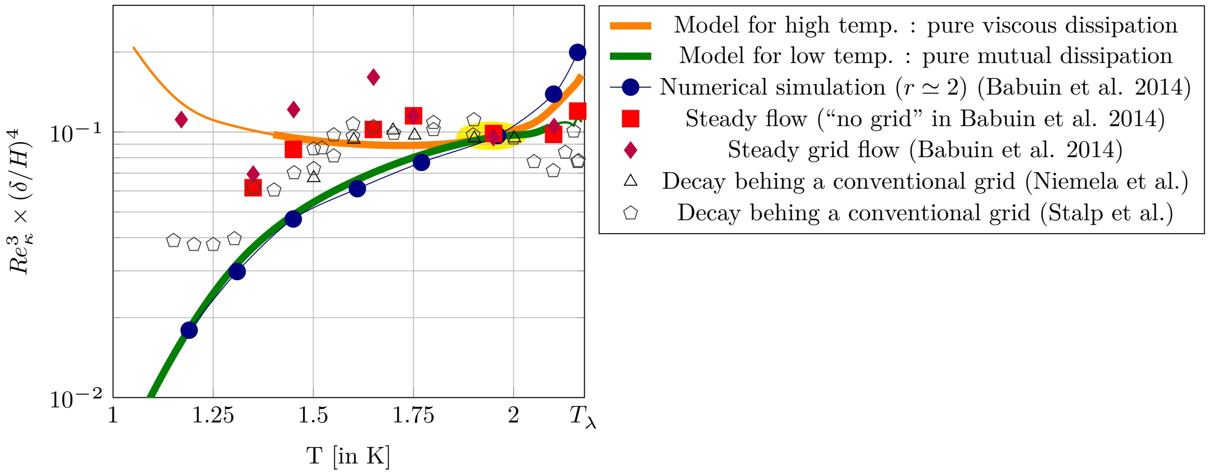

Figure 1. Superfluid Schmidt number Sc versus temperature. An arbitrary offset has been applied to each datasets (see [2])

done on figure 1 with the data from Niemela and Stalp, see [2] for references), but this last equation implied some hypothesis about the decay. Thus, vortex line density measurement in steady-state mechanically driven turbulent superfluid flow are very valuable as direct test the relation between δ/H and $Re_\kappa^{-3/4}$ but these are scarce in the literature.

To complement the numerics and experiments in our 2014 paper, we will present an analysis of the datasets reported in the literature by Laguna in the far fields of a counterflow jet [3] , by Holmes and Van Sciver in a forced pipe flow [4] and by Stalp right behind a grid pulled at different velocities [5]. We will show that some estimations of Sc versus temperature can be derived from these experiments, although none meets ideal conditions to determine Sc. The resulting Sc will be compared to our 2014 study.

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