PRANDTL NUMBER EFFECTS ON THE DECAYING AND THE FORCED TURBULENCE IN STRATIFIED FLUIDS

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Abstract Effects of high-Prandtl number density-stratifying scalar, i.e., active scalar, on decaying and forced turbulence in stratified fluids are investigated by numerical simulations. In decaying turbulence, potential energy spectrum of the high-Prandtl number active scalar ($Pr = 6$) agrees with the kinetic energy spectrum even at small scales. In forced steady turbulence, these two spectra again approach each other at small scales. These phenomena, which are in disagreement with the Batchelor scaling [1] for a high-Schmidt number passive scalar, occur at scales even smaller than the Ozmidov scale, suggesting that these effects would not be negligible in general.

We consider a fluid containing two scalars T_* and S_*, which have different Prandtl numbers $Pr_T = \kappa_{T*}/\nu_* = 1$ and $Pr_S = \kappa_{S*}/\nu_* = 6$ (κ_{T*}, κ_{S*}: diffusion coefficients, ν_*: kinematic viscosity, and dimensional quantities are denoted by asterisks.) in contrast to many previous studies [2] ,[3], [4]. These two scalars have constant mean vertical gradients $N_{T*} \equiv d\bar{T}_*/dz_*(< 0)$ and $N_{S*} \equiv d\bar{S}_*/dz_*(< 0)$, where z_* is the vertical coordinate and the undisturbed mean density is given by $\bar{\rho}_*(z_*) = \rho_{0*}(1 + \alpha_* N_{T*} z_* + \beta_* N_{S*} z_*)$. In this study, however, only one of the scalar (T_* or S_*) is an active scalar and the other is a passive scalar, so that either α_* or β_* is zero.

Both the decaying and the forced turbulence are investigated in this study by direct numerical simulations. The governing equations are solved by the pseudo-spectral method with the resolution of 256^3 for the periodic cubic region with the domain size of 6π.

In decaying turbulence, we initially give an isotropic velocity fluctuations, whose rms velocity and integral length scale are given by U_{0*} and L_{0*}. If T_* is an active scalar, the Navier-Stokes equations under Boussinesq approximation and the convection equations for two scalars are given by (cf. [5])

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j^2} - \frac{1}{Fr^2} T\delta_{i3}, \tag{1}$$

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \frac{1}{RePr_T}\frac{\partial^2 T}{\partial x_j^2} + u_3 \quad \text{and} \quad \frac{\partial S}{\partial t} + u_j \frac{\partial S}{\partial x_j} = \frac{1}{RePr_S}\frac{\partial^2 S}{\partial x_j^2} + u_3, \tag{2}$$

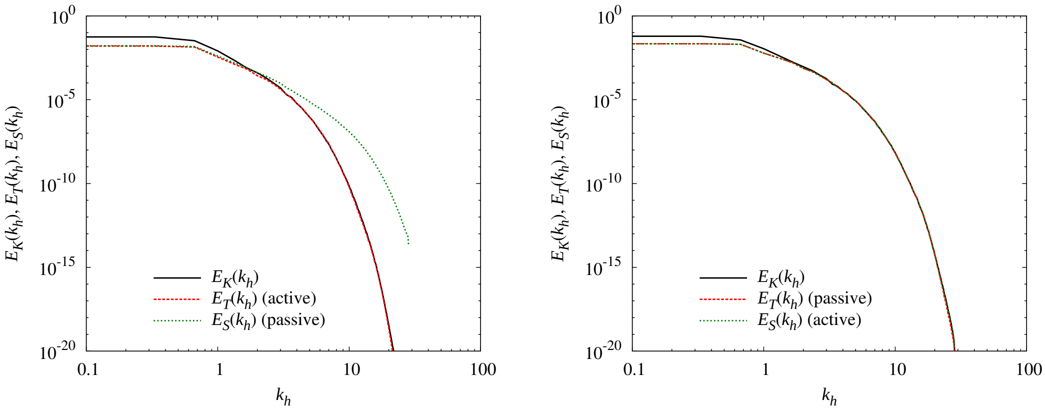
in which physical quantities are non-dimensionalized by the length scale L_{0*} and the velocity scale U_{0*}, T and S represent scalar perturbations, and $i = 3$ denotes the vertical component. The Reynolds number and the Froude number are defined by $Re = U_{0*}L_{0*}/\nu_*$ and $Fr = U_{0*} * /(N_* L_{0*})$, where N_* is the Brunt-Väisälä frequency.

On the other hand, in forced turbulence, external forcing injects energy randomly only into the horizontal modes ($k_3 = 0$) of horizontal velocity components (cf. [6]). The energy injection rate is fixed at P_*, and the length scale of forcing is characterized by the forcing wavenumber k_{f*}. Non-dimensionalization using the length scale $L_* = 1/k_{f*}$ and the velocity scale $U_* = (P_*/k_{f*})^{1/3}$, and the definition of the Reynolds number by $Re = U_* L_*/\nu_* = P_*^{1/3}/(k_{f*}^{4/3}\nu_*)$ and the Froude number by $Fr = U_*/(N_* L_*) = P_*^{1/3} k_{f*}^{2/3}/N_*$ lead to the same governing equations as (1) and (2), except for the forcing term.

We first show the results of decaying turbulence with $Re = 90$ and $Fr = 0.2$, for which the initial Taylor micro-scale Reynolds number is $Re_\lambda \sim 50$. Figure 1a and b present the horizontal spectra of velocity and two scalars when only low-Pr scalar $T(Pr = 1)$ is an active scalar (figure 1a), and when only high-Pr scalar $S(Pr = 6)$ is an active scalar (figure 1b). In figure 1a, spectrum of the passive scalar S with high $Pr(= 6)$ has large fluctuations at small scales, consistent with the Batchelor scaling [1], while the spectrum of active scalar $T(Pr = 1)$ almost agrees with that of kinetic energy as can be easily expected. On the other hand, in figure 1b, the spectra obey the Batchelor scaling only very initially before the buoyancy force becomes effective, and the small scale fluctuations of high-Pr active scalar $S(Pr = 6)$ rapidly decrease to finally agree with the spectra of the kinetic energy and the other passive scalar $T(Pr = 1)$, which are at higher levels than in figure 1a. These results show the deviation from the usual Batchelor scaling for a high-Prandtl number active scalar. Examination of the spectrum of the vertical flux shows that the potential energy of high-Pr scalar is persistently converted into the vertical kinetic energy through the negative, i.e., counter-gradient, vertical density flux.

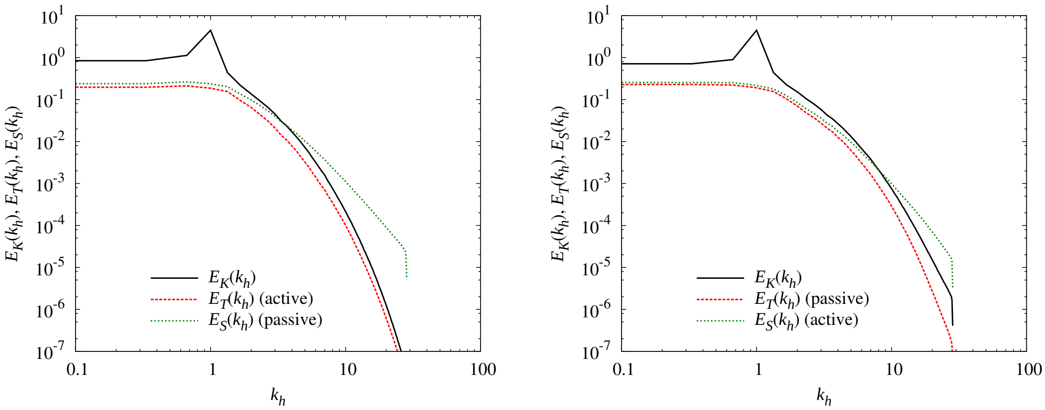
We next show the results for the forced (steady) turbulence with $Re = 40$, $Fr = 0.2$, for which the Taylor micro-scale Reynolds number is $Re_\lambda \sim 80$. Figure 2 a and b display the horizontal spectra when only the low-Pr scalar $T(Pr = 1)$ is an active scalar (figure 2a), and when only the high-Pr scalar $S(Pr = 6)$ is an active scalar (figure 2b). We observe in figure 2a that the passive scalar $S(Pr = 6)$ has the largest fluctuations at small scales as in the decaying turbulence (cf. figure 1a). On the other hand, in figure 2b, the small-scale fluctuations of high-Pr active scalar $S(Pr = 6)$ somewhat decreases to approach the spectra of kinetic energy and the other passive scalar $T(Pr = 1)$, which are at higher levels than in figure 2a. Although the agreement of three spectra is not so complete as in the decaying turbulence, qualitative trend is similar in the decaying and the forced turbulence. Namely, the potential energy spectrum of the high-Pr active scalar and the kinetic energy spectrum tends to agree, even at small scales.

We should finally mention that this tendency to *spectral agreement* occurs even at scales smaller than the Ozmidov scale where the buoyancy effects would be small. For example, in figure 2, the Ozmidov scale is at $k_h \sim 13$, while the tendency to agreement is observed even at much higher wave numbers.



(a) (b)

Figure 1. Horizontal spectra of kinetic energy and two scalars in decaying turbulence at $t = 37.5$, with initial values of $Re = 90$ and $Fr = 0.2$. Spectra are defined by $KE = \overline{u_i^2}/2 = \int E_K(k_h)dk_h, \overline{T^2}/(2Fr^2) = \int E_T(k_h)dk_h$ and $\overline{S^2}/(2Fr^2) = \int E_S(k_h)dk_h$, where overlines denote the average in space. (a) $T(Pr = 1)$: active, $S(Pr = 6)$: passive; (b) $S(Pr = 6)$: active, $T(Pr = 1)$: passive.



(a) (b)

Figure 2. Horizontal spectra of kinetic energy and two scalars in forced steady turbulence with $Re = 40, Fr = 0.2$. The spectra are averaged over the period of $45 \leq t \leq 75$. The spectral peak at the wavenumber of 1 corresponds to the forcing wavenumber. (a) $T(Pr = 1)$: active, $S(Pr = 6)$: passive; (b) $S(Pr = 6)$: active, $T(Pr = 1)$: passive.

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