INVARIANT SOLUTIONS IN LARGE EDDY SIMULATION OF HOMOGENEOUS SHEAR TURBULENCE

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Abstract The unstable invariant solutions in the large eddy simulation of homogeneous shear turbulence with vanishing kinematic viscosity are obtained by Newton-Krylov-hookstep method. The small scale is represented by the standard Smagorinsky model with a constant C_s. It is shown that these solutions appear by a saddle-node bifurcation as decreasing C_s and have the same symmetry with Nagata's equilibrium solution in Couette flow (*JFM* 217, 519-527 (1990)). Both lower- and upper- branch solutions are characterized by staggered streamwise-inclined vortex pairs. Also, lower-branch solutions are localized in the vertical direction, while upper-branch solutions are characterized by taller flow structures, which is consistent with the asymptotic theory of any shear flow at high-Reynolds numbers (K. Deguchi & P. Hall, *Phil. Trans. R. Soc A*, 372:20130352 (2014)).

INTRODUCTION

It has been known that coherent structures in turbulence are incomplete representations of unstable equilibrium solutions or periodic orbits in the Navier-Stokes equations [5, 4]. These invariant solutions have advantages to elucidate the dynamics of turbulence, since they are reproducible. At high Reynolds numbers, however, these solutions are hard to be tracked as a function of the Reynolds number, and their relevance to fully-developed turbulence are not revealed yet. The key idea is to model the small scale dynamics and to focus on large-scale motions, as previous works [3, 8], which will reduce the size of the nonlinear systems that we need to solve to obtain an invariant solution. The main purpose of this study is to capture such invariant solutions in the large eddy simulations (LES) of homogeneous shear turbulence (HST) with an eddy viscosity model (here we use the standard Smagorinsky model).
The filtered incompressible Navier-Stokes equation and continuity equations

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[2(\nu+\nu_t)\overline{S}_{ij}\right], \quad \frac{\partial \overline{u_j}}{\partial x_j} = 0, \tag{1}$$

where $\overline{u}_i$, $\overline{p}$, ν and ν_t are the filtered velocity, modified pressure (including the diagonal part of the subgrid-scale stress tensor), kinematic viscosity ($\nu = 0$) and eddy viscosity $\nu_t \equiv (C_s\Delta)^2(2\overline{S}_{ij}\overline{S}_{ij})^{1/2}$ ($\overline{S}_{ij}$ is the grid-scale strain-rate tensor and $\Delta \equiv \sqrt[3]{\Delta x \Delta y \Delta z}$ is the grid-scale filter size), are formulated in terms of the vertical vorticity and of the Laplacian of the vertical velocity [2]. The computational domain is periodic in the streamwise (x) and spanwise (z) directions, and periodic between shifting points of the lower and upper boundaries (so-called "shear-periodic" boundary condition). The discretization is the dealiased Fourier expansion in (x, z), and compact finite differences in the vertical direction (y), with the shear-periodic boundary conditions embedded in the finite-difference matrices for each Fourier mode [6]. The nondimensional parameters are the two aspect ratios of the computational box, $A_{xz} = L_x/L_z$ and $A_{yz} = L_y/L_z$ and the Smagorinsky constant C_s, where L_x, L_y and L_z are streamwise, vertical and spanwise length of the computational domain. The physical grid points are $(64, 48, 32)$. In this study, the solutions are obtained by Newton-Krylov-hookstep method [7] in the symmetric subspace: (I) a reflection with respect to the plane of $z = 0$ plus a streamwise shift by $L_x/2$, and (II) a rotation by π around the line $x = y = 0$ plus a spanwise shift by $L_z/2$:

$$\text{(I) } [u,v,w](x,y,z) = [u,v,-w](x+L_x/2, y, -z); \quad \text{(II) } [u,v,w](x,y,z) = [-u,-v,w](-x,-y,z+L_z/2). \tag{2}$$

The lower- and upper-branch solutions are continued along C_s by using the arc-length method.

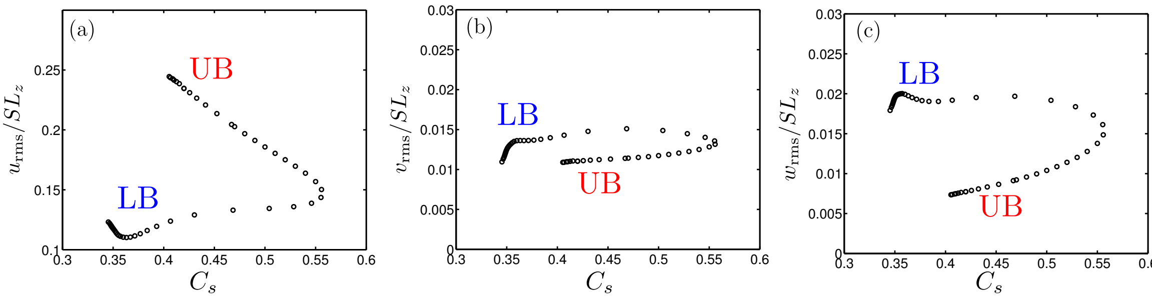

Figure 1. The box-averaged velocity fluctuations normalized by SL_z as a function of the Smagorinsky constant C_s for the box-aspect ratios $(A_{xz}, A_{yz}) = (3, 1.33)$. (a) $u_{\rm rms}/SL_z$; (b) $v_{\rm rms}/SL_z$; (c) $w_{\rm rms}/SL_z$.

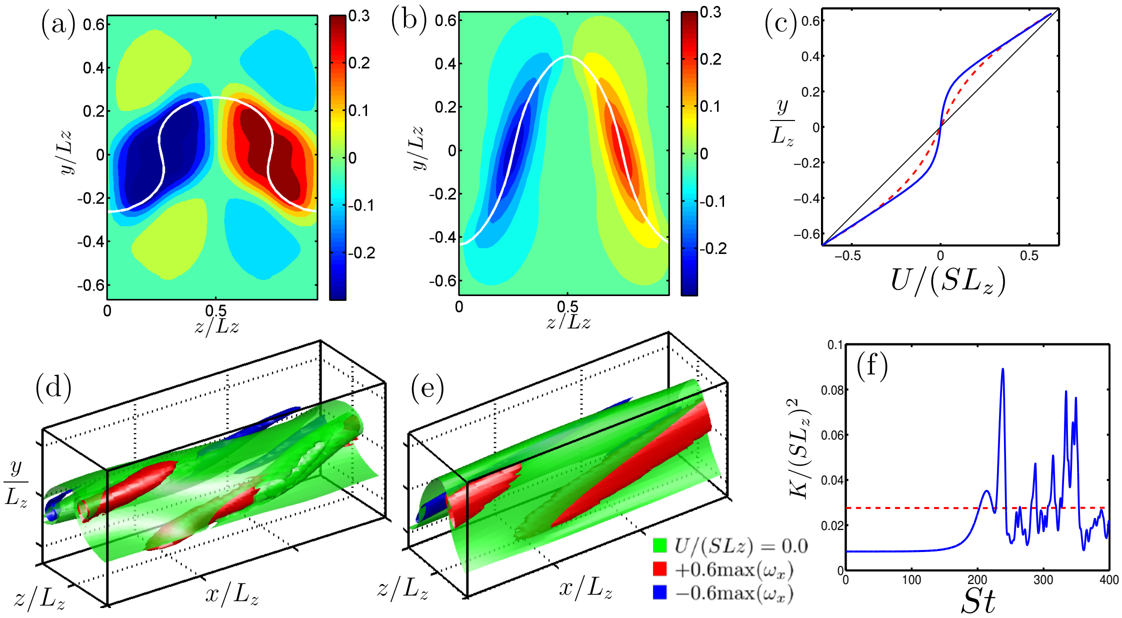

Figure 2. The flow structure of the lower- (a,d) and upper-branch (b,e) solution for $(A_{xz}, A_{yz}) = (3, 1.33)$ and $C_s = 0.42$. (a,b) The streamwise-averaged $\omega_x(y,z)/S$ (contours), and the total streamwise velocity $U/L_z = 0.0$ (white solid line); (c) the mean streamwise velocity of the lower- (blue thick line) and upper-branch solution (red dashed line), the black thin line is Sy/L_z; (d,e) isosurfaces of $|\omega_x| = 0.6|\omega_x|_{\max}$ (red, blue) and $U/L_z = 0.0$ (green), representing 3d-vortical structures and streaks of the lower- (b) and upper-branch (d) solutions. (f) The time-evolution of kinetic energy K from the lower-branch solution (blue). The red dashed line represents K of the corresponding upper-branch level. Values are normalized by $(SL_z)^2$.

RESULTS

Figure 1 shows the box-averaged velocity fluctuations continued along the Smagorinsky constant C_s of the lower- and upper-branch solutions (LB, UB, hereafter). LB is characterized by the weaker streamwise velocity fluctuation and slightly larger cross-streamwise velocity fluctuations. As shown in Fig. 2 (a,b,c), the LB solution is localized in y direction (at around $y = 0$ because of the symmetry), which may be represented by the asymptotic theory of the exact coherent structure [1]. Figure 2 (d,e) shows isosurfaces of $|\omega_x| = 0.6|\omega_x|_{\max}$ and $U/L_z = 0.0$, representing 3d-vortical structures and streaks of LB and UB solutions at $C_s = 0.42$. The streamwise-velocity streak of the LB solution meanders more than that of UB, on the other hand, the UB solution is characterized by the taller structure of the streak and the vortical structures. The time-evolution from a LB solution with $C_s = 0.42$ is performed, being restricted in the above symmetric subspace, as shown in Fig. 2(f). It exhibits that the kinetic energy bursts-up toward the level of the UB solution, and later, the flow represents the chaotic behaviour. In ongoing work, we are tracking these solutions up to $C_s = 0.1 - 0.2$ which are the standard values for the LES of homogeneous shear turbulence to represent turbulence velocity spectra. Also, these LES solutions will be compared with the previously computed unstable periodic orbits in the direct numerical simulation of HST.

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References

[1] K. Deguchi and P. Hall. Canonical exact coherent structures embedded in high reynolds number flows. *Phil. Trans. R. Soc. A*, **372**:20130352, 2014.

[2] T. J. R. Hughes, A. A. Oberai, and L. Mazzei. Large eddy simulation of turbulent channel flows by the variational multiscale method. *Phys. Fluid*, **13**:6, 2001.

[3] Y. Hwang and C. Cossu. Self-sustained process at large scales in turbulent channel flow. *Phys. Rev. Lett.*, **105**:044505, 2010.

[4] Genta Kawahara, Markus Uhlmann, and Lennaert Van Veen. The significance of simple invariant solutions in turbulent flows. *Ann. Rev. of Fluid Mech.*, **44**:203–225, 2012.

[5] M. Nagata. Three-dimensional finite-amplitude solutions in plane Couette flow: bifurcation from infinity. *J. Fluid Mech.*, **217**:519–527, 1990.

[6] A. Sekimoto, S. Dong, and J. Jiménez. Direct numerical simulation of statistically stationary homogeneous shear turbulence. *J. Comput. Phys. (in preparation)*.

[7] D. Viswanath. Recurrent motions within plane Couette turbulence. *J. Fluid Mech.*, **580**:339–358, 2007.

[8] T. Yasuda, S. Goto, and G. Kawahara. Quasi-cyclic evolution of turbulence driven by a steady force in a periodic cube. *Fluid Dyn. Res.*, **46**:061413, 2014.