EXPLICIT ALGEBRAIC AND DIFFERENTIAL REYNOLDS STRESS MODEL APPLICATION TO HOMOGENEOUSLY SHEARED AND COMPRESSED TURBULENCE.

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Abstract An explicit algebraic and differential Reynolds stress models (EARSM and DRSM) are used to investigate the influence of homogeneous shear and compression on the behaviour of turbulence in the limit of rapid distortion theory (RDT). EARSM is shown to give realizable results and to preserve RDT regime, unlike the eddy-viscosity model (EVM). The DRSM version of our model is in reasonable agreement with RDT theory.

EXPLICIT ALGEBRAIC REYNOLDS STRESS MODEL

In (1) we developed an explicit algebraic Reynolds stress model for compressible turbulent flows taking into account strong dilatation of the flow. In a two-dimensional case the model for Reynolds stress anisotropy tensor $\mathbf{a}$ is given by a formula

$$\mathbf{a} = -\frac{6}{5}\,\frac{1}{N^2 - 2\,\mathit{II}_{\boldsymbol{\Omega}}}\Big(N\,\mathbf{S}^{2D} + \mathbf{S}^{2D}\,\boldsymbol{\Omega} - \boldsymbol{\Omega}\,\mathbf{S}^{2D}\Big) - \frac{3}{5}\,\frac{\mathcal{D}}{N}\Big(\boldsymbol{\delta}^{2D} - \frac{2}{3}\,\boldsymbol{\delta}^{3D}\Big),\quad \mathbf{S}^{2D} = \begin{pmatrix} \mathcal{J} & \sigma \\ \sigma & -\mathcal{J}\end{pmatrix},$$
$$\boldsymbol{\Omega} = \begin{pmatrix} 0 & \omega \\ -\omega & 0\end{pmatrix},\quad \mathcal{D} = \tau\,(\partial_x\,U_x + \partial_y\,U_y),\quad \mathcal{J} = \frac{\tau}{2}\,(\partial_x\,U_x - \partial_y\,U_y),\quad \sigma/\omega = \frac{\tau}{2}\,(\partial_y\,U_x \pm \partial_x\,U_y), \tag{1}$$

where $\boldsymbol{\delta}^{2D}$ and $\boldsymbol{\delta}^{3D}$ are two-dimensional and three-dimensional Kronecker tensors, respectively, $\mathbf{S}^{2D}$ and $\boldsymbol{\Omega}$ – two-dimensional strain-tensor and vorticity-tensor, respectively, τ – turbulence time scale. Parameter N is determined from the solution to a quartic equation which depends on a model constant c_1', the dilatation $\mathcal{D}$ and invariants of the flow $\mathit{II}_S = 2\,(\sigma^2 + \mathcal{J}^2) + \mathcal{D}^2/6$ and $\mathit{II}_{\boldsymbol{\Omega}} = -2\,\omega^2$

$$N^4 - c_1'\,N^3 - \Big(2\,\mathit{II}_{\boldsymbol{\Omega}} + \frac{27}{10}\,\mathit{II}_S\Big)\,N^2 + 2\,c_1'\,\mathit{II}_{\boldsymbol{\Omega}}\,N + \frac{9}{10}\,\mathit{II}_{\boldsymbol{\Omega}}\,\mathcal{D}^2 = 0. \tag{2}$$

Here we subject the flow to rapid homogeneous shear and compression. The evolution of the mean quantities depends on the parameters s_0 and $d_0 < 0$:

$$U_x = \Big(1 + d_0\,t\Big)^{-1}\Big(d_0\,x + s_0\,y\Big),\quad U_{y,z} = 0,\quad \rho = \Big(1 + d_0\,t\Big)^{-1}\rho_0,\quad P = \Big(1 + d_0\,t\Big)^{-\gamma} P_0. \tag{3}$$

This identically fulfills the equations of motion of a flow with low level of turbulence kinetic energy:

$$D_t\,U_i + \partial_i\,P \equiv 0,\quad \partial_t\,\rho + \partial_k\,(\rho\,U_k) \equiv 0,\quad D_t\,P + \gamma\,P\,\partial_k\,U_k \equiv 0. \tag{4}$$

THE RESULTS

We investigate the behaviour of the turbulence with our EARSM complemented by a $k - \omega$ model (1) in the RDT limit. Fig.1 shows the evolution of the components of production, anisotropies, turbulence kinetic energy, 'turbulence frequency' ω and 'total strain-rate' $S^* = \sqrt{2\,(\mathit{II}_S + \mathcal{D}^2/3)}$. The last quantity is provided to check if we are in the RDT regime ($S^* \gtrsim 2$). The cases plotted comprise the magnitudes of 'shear-compression' ratio (only the ratio is physically important in the RDT limit) $s_0/d_0 = -2.0, -1.0$ and -0.3. For all three cases the model is realizable and sustains the regime of the applicability of RDT. The last condition may be violated when $|s_0/d_0|$ exceeds ~ 10. By contrast, in the eddy-viscosity model $a_{ij} = -2\,\beta_s\,S_{ij}$ the total-strain S^* quickly becomes less than ~ 2, *i.e.* the RDT-regime is not captured by the EVM. The EVM also becomes unrealizable when $|s_0/d_0| \lesssim 1$.
In (2) the authors investigated the influence of homogeneous compression on turbulence which has been subjected to a certain amount of homogeneous shear in advance. Here we use the DRSM of the EARSM (they employ the same modeling of the pressure-strain terms but DRSM does not use the weak-equilibrium assumption and therefore reacts gradually to a change in flow conditions) to compare with RDT the result of applying

a homogeneous compression to the turbulence. We assume that the total amount of shear is $s_0 t = 3$ and take the corresponding initial values of anisotropies from RDT. The evolution of anisotropies, K and ω under homogeneous compression with $s_0/d_0 = -0.1$ is plotted in fig.2. The DRSM and RDT give the same trends and are in good agreement for K.

The EARSM presented has also been generalized to account for the influence of turbulent density fluxes of a turbulent flow (3). In principle the model can be used to investigate the effect of the interaction of the density and pressure gradients in the RDT regime. This aspect will be further investigated.

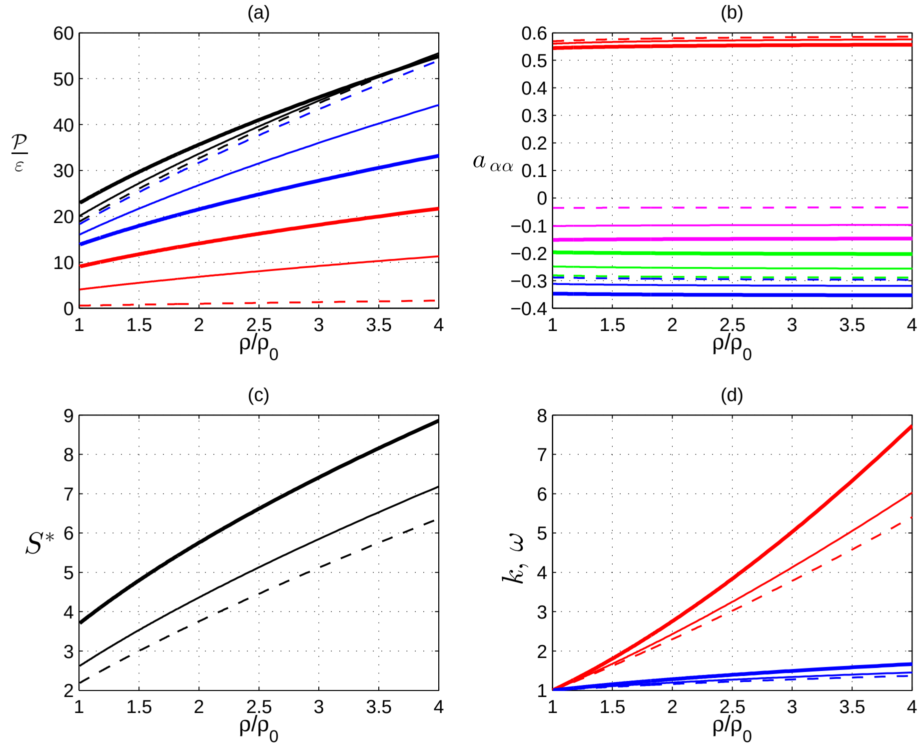

Figure 1. (a) – $\mathcal{P}/\varepsilon$: red shows 'shear' part, blue – 'dilatational' part, black – totat production. (b) – anisotropy tensor: a_{11} – red, a_{22} – blue, a_{33} – green, a_{12} – magenta. (c) total shear. (d) – red – K, blue – ω. Thick solid lines, thin solid lines and dashed lines denote the cases with $s_0/d_0 = -2.0, -1.0, -0.3$, respectively.

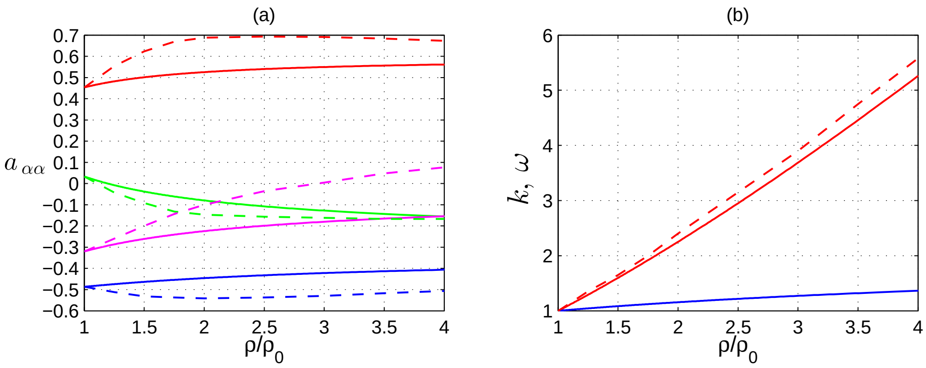

Figure 2. (a) – anisotropies, (b) – K and ω (see fig.1 for designations). Solid lines – DRSM, dashed lines – RDT.

References

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