ANALYSIS OF TURBULENT RAYLEIGH-BÉNARD CONVECTION IN THE COMPOUND PHYSICAL/SCALE SPACE DOMAIN

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Abstract We report the results from two distinct direct numerical simulations (DNS) of turbulent Rayleigh-Bénard convection (RBC) for Rayleigh number of 10^5 and Prandtl number of 0.7 in a laterally unbounded domain confined between two horizontal isothermal plates with no-slip and free-slip boundary conditions respectively. The central aim of the present work consists in a simultaneous description of both flows in a compound physical/scale space domain by using a generalized form of the classical Kolmogorov equation for the second-order velocity structure function. It has been found that the dynamics of the coherent structures in RBC, the so-called thermal plumes, are clearly reflected in the multi-scale energy budgets. In particular, the enlargement of thermal plumes following the impingement at the wall entails a transfer of scale-energy from small turbulent scales toward larger ones. This aspect shed light on the role of thermal plumes in turbulent RBC and could have a direct impact on future attempts to model the effects of small-scale motions in thermal convection.

ANALYSIS OF THE FLOW TOPOLOGY

It is well known that the most prominent structures in turbulent RBC are the so-called thermal plumes, which can be defined as localized portions of fluid having a temperature contrast with the background [1]. Hot and cold plumes detach respectively from the lower and the upper wall, stretch across the domain and, finally, enlarge close to the opposite plate. Furthermore these structures have a sheet-like form at the beginning, whereas they take the appearance of a mushroom sufficiently away from the starting point [2]. By considering the two-dimensional divergence of the velocity field in the xy-plane, it is possible to measure both the horizontal enlargement and narrowing of thermal plumes [3]. Figure 1(a) and (b) show an hot isosurface of temperature ($\theta = 0.25$) coloured with the horizontal divergence of the velocity field $div_\pi = \partial u/\partial x + \partial v/\partial y$ for the turbulent RBC with no-slip and free-slip boundary conditions respectively. In both cases, two distinct events can be identified in terms of the horizontal divergence: the ejection ($div_\pi < 0$) and the impingement of thermal plumes ($div_\pi > 0$). Adjacent regions of positive divergence are separated by thin filaments having a negative divergence, hence the sheet-like roots protrude from the thermal boundary layer mainly as a consequence of the mechanical action of impinging plumes. Furthermore, the intersection of different sheet-like roots leads to a large concentration of momentum, which is in turn responsible, together with the buoyancy forces, for the ejection of new structures. In Figure 1(c) the quantities $\langle div_\pi \rangle_+$ and $-\langle div_\pi \rangle_-$, where $\langle div_\pi \rangle_+ = \langle div_\pi \rangle$ for $div_\pi > 0$ and $\langle div_\pi \rangle_- = \langle div_\pi \rangle$ for $div_\pi < 0$, are plotted. These conditional statistics allow us to display the magnitude of both the impingement ($\langle div_\pi \rangle_+$) and the ejection ($\langle div_\pi \rangle_-$) as a function of the distance from the wall $z^* = 0.5 - |z|$. As can be expected, the plume dynamics changes deeply from the free-slip case to the no-slip one especially close to the walls. In particular, the peaks of both impingement and ejection in the free-slip flow are located at $z^* = 0$, see the inset of Figure 1(c), whereas in the no-slip one the impingement reaches its maximum at $z^* = 0.07$ and the ejection is peaked slightly further away at $z^* = 0.10$, see the main plot in Figure 1(c). The different interaction of thermal plumes with the vertical boundaries is a discriminating factor in turbulent RBC dynamics which can be analyzed successfully in terms of multi-scale energetics.

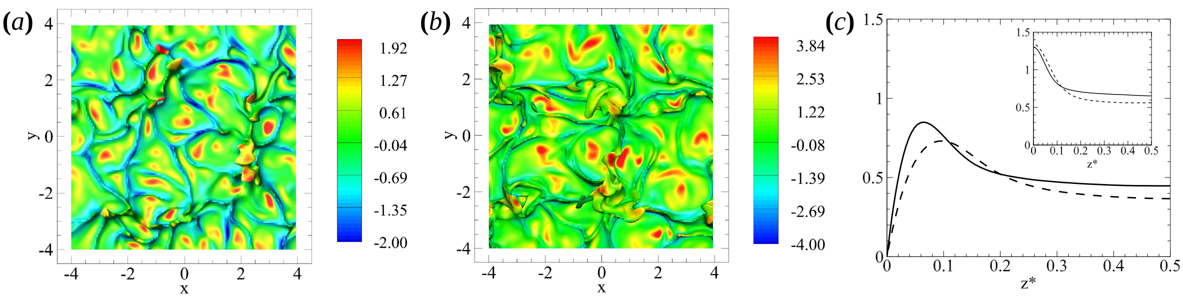

Figure 1. (*a*) Top view of the isosurface of temperature at $\theta = 0.25$ for the no-slip and (*b*) the free-slip flows coloured by the horizontal divergence div_π. (*c*) Main plot: $\langle div_\pi \rangle_+$ (solid line) and $-\langle div_\pi \rangle_-$ (dashed line) vs. z^* for the no-slip case. Inset: as the main plot but for the free-slip case.

MULTI-SCALE ANALYSIS OF THE FLOW

The production, transport and dissipation of velocity fluctuations depend both on the geometrical location within the flow and on the turbulent scale considered i.e. they are inherently multi-scale. In this scenario, a compound description in the physical/scale space is required to understand the physics of turbulent convection [4]. An appropriate candidate to consider for a simultaneous description of turbulent dynamics in physical and scale space is the second-order velocity structure function $\langle \delta u^2 \rangle = \langle (u_i(x_i + r_i) - u_i(x_i))^2 \rangle$, where x_i is the position vector, r_i is the separation one and u_i is the velocity field. Hereafter, $\langle \delta u^2 \rangle$ will be referred to as scale-energy, indeed it can be considered as a roughly measure of the kinetic energy at scale $|r_i|$. The budget for $\langle \delta u^2 \rangle$ is referred to as generalized Kolmogorov equation and it represents an extension to an inhomogeneous flow of the balance proposed by Kolmogorov [6] for homogeneous and isotropic turbulence. It can be obtained following the procedure described in [5] and corresponds to

$$\frac{\partial \langle w^* \delta u^2 \rangle}{\partial Z_c} + 2\frac{\partial \langle \delta p \delta w \rangle}{\partial Z_c} - \frac{1}{2}\sqrt{\frac{Pr}{Ra}}\frac{\partial^2 \langle \delta u^2 \rangle}{\partial Z_c^2} + \frac{\partial \langle \delta u^2 \delta u_i \rangle}{\partial r_i} - 2\sqrt{\frac{Pr}{Ra}}\frac{\partial^2 \langle \delta u^2 \rangle}{\partial r_j \partial r_j} = 2\langle \delta\theta \delta w \rangle - 4\langle \epsilon^* \rangle\,, \tag{1}$$

where the asterisk denotes the mid-point average, $\beta^* = (\beta(x_i) + \beta(x_i + r_i))/2$ for a generic quantity β. Here, p is the pressure field, θ is the temperature one, w is the velocity component u_3 and $\langle \epsilon \rangle$ is the average rate of pseudo-dissipation. The balanced terms depend on the separation vector r_i and on the geometrical coordinate $Z_c = z + r_z/2$, where $r_z = r_3$. The first, the second and the third terms on the left hand side are respectively the inertial, the pressure and the viscous contributions to the transport in physical space, whereas the fourth and the fifth terms are respectively the inertial and the viscous contributions to the transfer in the scale space. The first and the second terms on the right hand side are the buoyant production and the viscous dissipation of scale-energy. The classic turbulent kinetic energy budget represents a limit case of the scale-energy budget, indeed equation (1) converge to four times the single-point budget at sufficiently large separations [7]. By considering $r_z = 0$ and by averaging equation (1) over a circle C of radius r, $\left(\pi r^2\right)^{-1} \int_{C(r)} dr_x dr_y$, we obtain the r-averaged scale-energy budget

$$I_c(r, Z_c) + P(r, Z_c) + D_c(r, Z_c) + I_r(r, Z_c) + D_r(r, Z_c) = \Pi(r, Z_c) + E(Z_c), \tag{2}$$

where each term corresponds to the appropriate term in equation (1). The buoyant production and the viscous dissipation can be condensed into a single term $S = \Pi + E$. Furthermore, by defining an overall contribution to the transport in physical space $T_c = I_c + P + D_c$ and an overall contribution to the transfer in the space of scales $T_r = I_r + D_r$, equation (2) can be rewritten in the compact form

$$T_c(r, Z_c) + T_r(r, Z_c) = S(r, Z_c). \tag{3}$$

It clearly emerges from equation (3) the role of the term S as a source of two distinct scale-energy fluxes: one in the physical space and the other in the scale space. The spatial flux induced by the vertical inhomogeneity modulates the scale-energy budget in such a way that a reverse flux of scale-energy in the space of scales (from small toward larger r) occurs inside a localized region of the $(r - Z_c)$-space. This particular phenomenon has never been observed nor investigated in turbulent RBC and could have strong repercussions on both theoretical and modeling approaches to convective turbulence. The reverse flux of scale-energy is found to be related with the dynamics of the coherent structures close to the walls in both the free-slip and the no-slip flows. In particular, the peak of the reverse flux is observed at the same distance from the wall as the maximum of $\langle div_\pi \rangle_+$, which measures the uttermost enlargement of thermal plumes due to the impingement.

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