LONGITUDINAL AND TRANSVERSE LAGRANGIAN VELOCITY INCREMENTS

Emmanuel Lévêque[1] & Aurore Naso[1]
[1]*Laboratoire de Mécanique des Fluides et d'Acoustique*
Ecole centrale de Lyon & CNRS, University of Lyon, France

Abstract Longitudinal and transverse Lagrangian velocity increments are introduced as components along, and perpendicular to, the displacement of fluid particles during a time scale τ. These increments provide a new path to the characterization of Lagrangian statistics in homogeneous and isotropic turbulence, and allows us to establish some bridge with Eulerian statistics. From direct numerical simulations, it is shown that the probability distributions of the longitudinal Lagrangian increment are negatively skewed at all time scales, which is a signature of time irreversibility in the Lagrangian framework. Transverse increments are found more intermittent than longitudinal increments. Eventually, transverse Lagrangian increments exhibit scaling properties that are very close to those of standard Cartesian Lagrangian increments.

Keywords : Fundamentals, Lagrangian dynamics, Scaling laws

The acceleration of a material point (or particle) is usually decomposed into a tangential and a normal component. The tangential acceleration quantifies the variation of the magnitude of the velocity (and therefore relates to the variation of kinetic energy of the particle) whereas the normal acceleration is sensitive to the curvature of the trajectory. It is natural to seek for a similar decomposition for the Lagrangian velocity increment $\delta\mathbf{u}^{(L)}(\mathbf{x},t|s) \equiv \mathbf{u}(\mathbf{x},t|s) - \mathbf{u}(\mathbf{x},t|t)$, where $\mathbf{u}(\mathbf{x},t|s)$ denotes the velocity at time s of a fluid particle that passed through the position $\mathbf{x}$ at time t. Accordingly, it is proposed to split $\delta\mathbf{u}^{(L)}(\mathbf{x},t|s)$ into a longitudinal and a transverse component, along and perpendicular to the direction indicated by the overall displacement of the fluid particle $\mathbf{y}(\mathbf{x},t|s) = \int_t^s \mathbf{u}(\mathbf{x},t|s')ds'$ during the time-lag $\tau = s - t$ (see Fig. 1). This splitting somewhat generalizes the decomposition of the (instantaneous) acceleration to the coarse-grained dynamics at time scale τ.

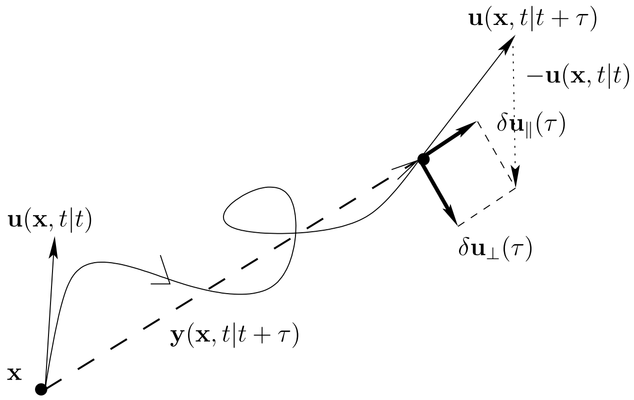

Figure 1. Sketch of the longitudinal and transverse Lagrangian velocity increments, along and perpendicular to, the direction given by the displacement vector.

These new Lagrangian increments exhibit striking features. In particular, it is observed that the PDFs of the longitudinal increment $\delta u_{\parallel}^{(L)}(\tau)$ are negatively skewed at all time scales τ (Fig. 2). This property can be related to the time irreversibility of the Lagrangian dynamics. Interestingly, in the inset of Fig. 2(a), $\langle \delta u_{\parallel}^{(L)}(\tau)^3\rangle/\varepsilon u_{\mathrm{rms}}\tau$ exhibits a plateau that would be reminiscent of the Kolmogorov's 4/5 law by assuming that $r \propto u_{\mathrm{rms}}\tau$ and that the Eulerian velocity field remains frozen during the particle displacement.
The transverse Lagrangian increment behaves differently. It is more intermittent than the longitudinal increment and behaves in a similar way as the standard Cartesian Lagrangian increment $\delta u_x^{(L)}$, as evidenced by comparing the flatness coefficients of the three increments (see Fig. 3). In the inset of Fig. 3, the local fourth-order (relative) scaling exponent $\widetilde{\zeta}_{4i} = \mathrm{d}\log\langle \delta u_i^{(L)}(\tau)^4\rangle / \mathrm{d}\log\langle \delta u_i^{(L)}(\tau)^2\rangle$ is plotted for the three Lagrangian increments. The transverse and Cartesian increments behave quite similarly and agree with the data reported in the review paper [2]. Nevertheless, the power-law scaling is more pronounced for the transverse increment with $\widetilde{\zeta}_{4\perp} = 1.59 \pm 0.02$ in excellent agreement with experimental data (for the Cartesian increment) at $R_\lambda = 1100$.

During this conference, we would like to introduce longitudinal and transverse Lagrangian velocity increments in a fluid-mechanical context, and to discuss their key features. We will show that these new increments allow us to establish some

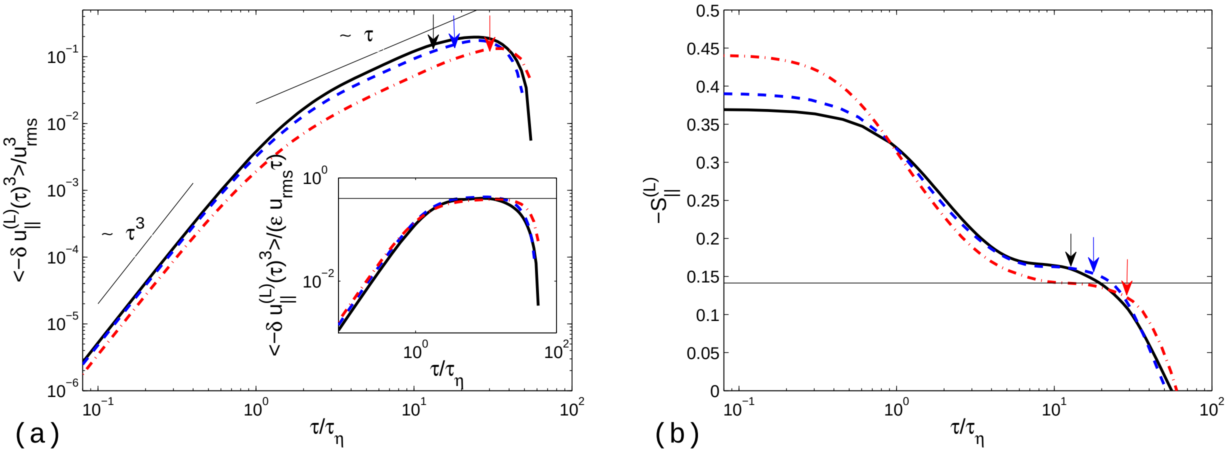

Figure 2. Dependence on τ of (a) the third-order moment of the longitudinal Lagrangian increment – Inset: compensated by $\varepsilon u_{\mathrm{rms}}\tau$ where ε is the mean dissipation rate and u_{rms} is the root-mean-squared velocity; (b) the skewness coefficient $S_{\|}^{(L)}(\tau) = \langle \delta u_{\|}^{(L)}(\tau)^3\rangle / \langle \delta u_{\|}^{(L)}(\tau)^2\rangle^{3/2}$. Solid line (black): $R_\lambda = 130$; dashed line (blue): $R_\lambda = 180$; dash-dotted line (red): $R_\lambda = 280$. Arrows mark the Lagrangian integral scales T_L, and τ_η denotes the Kolmogorov's time scale.

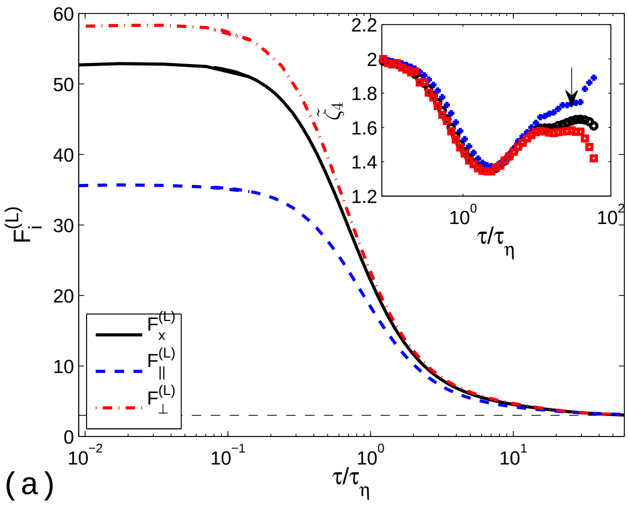

Figure 3. (a) Flatness of the different Lagrangian increments versus τ at $R_\lambda = 280$ (the dashed line indicates the value 3 for a Gaussian distribution) — Inset: Local fourth-order (relative) scaling exponent $\widetilde{\zeta}_{4i} = \mathrm{d}\log\langle \delta u_i^{(L)}(\tau)^4\rangle / \mathrm{d}\log\langle \delta u_i^{(L)}(\tau)^2\rangle$. Squares: transverse increments; Circles: Cartesian increments ; Crosses: longitudinal increments.

bridge with Eulerian increments, and open a new path for the characterization of Lagrangian statistics in turbulence. These results have been published in a recent paper [1].

References

[1] E. Lévêque, and A. Naso. Introduction of longitudinal and transverse Lagrangian velocity increments in homogeneous and isotropic turbulence. *Europhysics Letters* **108** (5): 54004, 2014.

[2] Arnéodo *et al.* Universal intermittent properties of particle trajectories in highly turbulent flows. *Phys. Rev. Lett.* **100** (25): 254504, 2008.