A LATTICE MODEL FOR THE EULERIAN DESCRIPTION OF HEAVY PARTICLES SUSPENSIONS IN ONE AND TWO DIMENSIONS

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Abstract Modeling of heavy particles motion in turbulent flows still represents a challenge in engineering applications at high Reynolds number. Various techniques have arisen for describing such mono-dispersed solid phases with statistical methods. Some of those techniques relies on the assumption of using a velocity field to describe the particles motion [1, 2], which is valid at small Stokes number, others using large-eddy simulations [4], or using one and two-points probability density functions in Gaussian flows [5]. Here we present another method based on a lattice discretization of the phase space in one and two dimensions for a synthetic flow in one dimension and a turbulent flow in two dimensions for the description of a dilute solid phase in the case of a Stokes coupling between the particles and the fluid and a brownian diffusion. This method is suited for any Stokes numbers in the limit of numerical stability and shows a good agreement with the Lagrangian particles statistics like radial distribution functions and collision rates.

INTRODUCTION

We present a method to simulate numerically the evolution of a collection of heavy particles in a random synthetic flow and a turbulent flow. The evolution of such particles can be described by the following equations :

$$\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} = \boldsymbol{V}(\boldsymbol{x},t) \quad \frac{\mathrm{d}\boldsymbol{V}(\boldsymbol{x},t)}{\mathrm{d}t} = \boldsymbol{\mathcal{F}}(\boldsymbol{X},\boldsymbol{V},t) + \boldsymbol{\eta}(t) \tag{1}$$

when submitted to a coupling $\boldsymbol{\mathcal{F}}$ with a liquid phase, which can *a priori* be any body force, and to brownian diffusion, with $\langle \eta_i(t)\, \eta_j(t') \rangle = 2\kappa\, \delta_{ij}\, \delta(t-t')$ a random force responsible for the diffusion, κ being the diffusion constant. The force $\boldsymbol{\mathcal{F}}$ exerted on the particles can be dissipative ($\nabla_{\boldsymbol{v}} \cdot \boldsymbol{\mathcal{F}} < 0$). We neglect particle-particle interactions. This is a rather good approximation for dilute suspensions.
A complete detail of the forces acting on particles in a flow can be found in [3] but we restrict to the Stokes drag $\boldsymbol{\mathcal{F}}(\boldsymbol{x},\boldsymbol{v},t) = (\boldsymbol{u}(\boldsymbol{x},t) - \boldsymbol{v})/\tau_p$, which is a valid form under the hypotheses of having a small particle Reynolds number, a small size compared to the smallest length scales of the flow and particles much heavier than the fluid. Deviations of the particles dynamic from the fluid flow are due to inertia which is measured in term of the response time τ_p (usually non-dimensionalised by a characteristic time of the flow to define the Stokes number St). By expressing the particles population with unitary mass in terms of a singular distribution $f(\boldsymbol{x},\boldsymbol{v},t) = \sum_p \delta(\boldsymbol{x}-\boldsymbol{x}_p)\delta(\boldsymbol{v}-\boldsymbol{v}_p)$ and taking the total time derivative, one can express the evolution equation for the distribution :

$$\partial_t f + \nabla_{\boldsymbol{x}} \cdot (\boldsymbol{v} f) + \nabla_{\boldsymbol{v}} \cdot (\mathcal{F}(\boldsymbol{x},\boldsymbol{v},t) - \kappa \nabla_{\boldsymbol{v}}) f = 0 \tag{2}$$

This is a conservation equation for an infinitesimal volume of particles flowing in the phase space. It can be seen as a Liouville equation or a collision-less Boltzmann equation, both stating that the density of points in the phase space along a trajectory evolved with the dynamical equations is constant with time.

METHOD

The equation (2) is discretized into finite volumes in the position-velocity phase space $(\boldsymbol{x}, \boldsymbol{v})$. Both the one dimensional and the two dimensional cases are studied.
In one dimension, a synthetic flow is used given by

$$u(\boldsymbol{x},t) = \phi_1(t)\cos(\boldsymbol{x}) + \phi_2(t)\sin(\boldsymbol{x}) \tag{3}$$

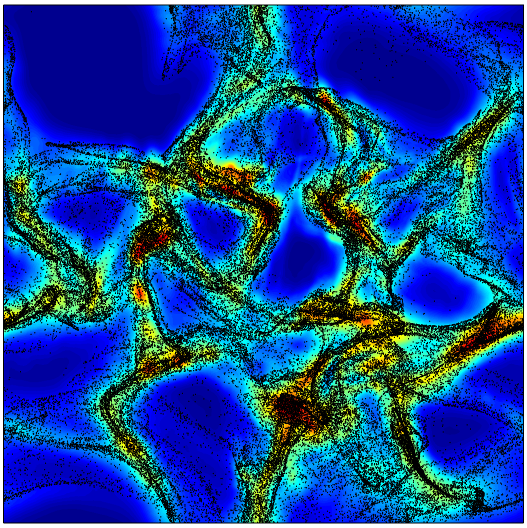
where ϕ_1 and ϕ_2 are Ornstein-Uhlenbeck processes.
In two dimensions, a pseudo-spectral Navier-Stokes solver is used to generate a direct enstrophy cascade with a large-scale random forcing.
In both cases, the position space is 2π-periodic in all directions and divided in N^d equally-spaced points, and the velocity space is divided into N_v^d cartesian volumes in the interval $[-u_{rms}, u_{rms}]$.

RESULTS

We show in figure 1 (left panel) an instantaneous field depicting the density of the Eulerian field in the position space defined as

$$\rho(\boldsymbol{r}) = \int_{\Omega_v} f(\boldsymbol{r}, \boldsymbol{v}) d\boldsymbol{v} \tag{4}$$

along with the particles at the same time. One can immediately see that the spatial distribution is correctly reproduced qualitatively. The regions where no or few particles are present, corresponding to high vorticity regions from which the heavy particles are ejected (see for example [2]) are shown to be indeed colored in deep blue indicating a quasi-null density of the Eulerian field.



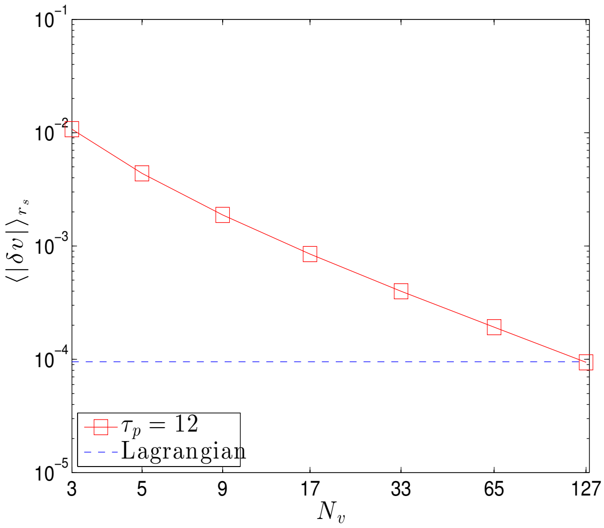


Figure 1. Left : A snapshot in the position space of a simulation in two dimensions. The color code is the density (defined in (4) of the Eulerian field, going from blue to red with increasing density. The black dots represent the particles. Right : Convergence of the average approach rate as a function of the velocity resolution of the Eulerian simulation in one dimension toward the actual Lagrangian statistics.

We then show in figure 1 (right panel) a more quantitative result for the one-dimensional case : the average approach rate of the particles which are separated by a given distance r_s, noted $\langle|\delta v|\rangle_{r_s}$. This is comparable to the structure function in dimension d, for which the velocity differences are taken along the vector joining the positions of the two particles, or the two points in space for the Eulerian field. In one dimension there is no need for such projection.
The approach rate shows a plateau for a certain range of scales (not shown here), and we compare the value of this plateau for various resolutions in the velocity space (expressed as the number of velocity values spanning the interval $[-u_{rms}, u_{rms}]$ to test the convergence to the corresponding Lagrangian statistic. The value of τ_p is taken to be 12 in this case, corresponding to $St = \frac{L/U_{rms}}{\tau_p} \approx 0.5$. The convergence is quite remarkable, although it needs a very fine resolution for the relative error to be acceptable.
To conclude, let us stress that such a novel method shows promising results for the Eulerian simulation of inertial particle dynamics. This opens the way to a different approach for subgrid-scale modeling of particle-laden turbulent flows.

References

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