ANALYSIS OF THE YAGLOM EQUATION AND SUBGRID MODELLING APPROACHES FOR THERMALLY DRIVEN TURBULENCE

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Abstract We report a Direct Numerical Simulation (DNS) of turbulent Rayleigh-Bénard convection in a laterally unbounded domain confined between two horizontal parallel walls, for Rayleigh number 10^5 and Prandtl number 0.7. The DNS data are used to study the properties of the subgrid-scale flux of the active temperature field in the framework of Large Eddy Simulation (LES). In particular, starting from the generalized Yaglom equation, we analyze how the thermal energy is produced, transferred and dissipated in the augmeneted space of scales and positions of the flow. The understanding of these processes is then used to propose appropriate formulations for the subgrid-scale flux that will be tested by means of *a posteriori* analysis of LES simulations performed in the same flow conditions.

INTRODUCTION

The large eddy simulation (LES) equations for turbulent Rayleigh-Bénard convection are obtained by applying a filtering operator to the continuity, momentum and temperature equations,

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0,$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \sqrt{\frac{Pr}{Ra}} \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} + \bar{\theta}\delta_{i3} - \frac{\partial \tau_{ij}}{\partial x_j},$$

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial \bar{u}_i \bar{\theta}}{\partial x_i} = \frac{1}{\sqrt{PrRa}} \frac{\partial^2 \bar{\theta}}{\partial x_i \partial x_i} - \frac{\partial \tau_{i\theta}}{\partial x_i},$$

where the overbar denotes filtered quantities, u_i and θ are the velocity and temperature fields and p is the pressure field. The effects of the subgrid-scales on the large resolved quantities appear in the subgrid stress tensor $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ and in the subgrid-scale flux $\tau_{i\theta} = \overline{u_i \theta} - \bar{u}_i \bar{\theta}$ that must be modelled. While there exists a large body of work on modeling τ_{ij}, relatively few closures are developed for $\tau_{i\theta}$ most of them are simply an adaptation of famous subgrid stress tensor models to the subgrid-scale scalar flux. The most popular models employ the eddy-viscosity approach [6], $\tau_{i\theta} = -(C_S\Delta)^2 |\bar{S}| \partial\bar{\theta}/\partial x_i$, where Δ is the characteristic length of the filter, $|\bar{S}| = \sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$ with $\bar{S}_{ij}$ the filtered strain rate tensor and C_S is the Smagorinsky constant. The main assumption of this modelling approach is that the subgrid scalar flux aligns with the gradient of the resolved scalar field. This property is overcomed by the scale-similarity model [1], $\tau_{i\theta} = \widetilde{\bar{u}_i \bar{\theta}} - \tilde{\bar{u}}_i \tilde{\bar{\theta}}$ where the tilde denotes the so-called test filter whose characteristic length is $\tilde{\Delta} > \Delta$. These two formulations are also coupled in the so-called mixed models [5]. In this class of models, a new formulation has been recently proposed for the subgrid stress tensor and it is found to properly capture the complex energetics of wall-bounded turbulence [3]. This model takes origin from a detailed analysis of the energy transfer in the augmented space of scales and position of the flow by means of the Kolmogorov equation generalized to inhomogeneous flows [4, 2]. Here, we extend this approach by analysing how thermal energy is produced, transferred and dissipated between different regions of the flow and scales by means of the Yaglom equation. Accordingly to these results, we extend the subgrid stress tensor model proposed in [3] taking into account a new formulation also for the subgrid scalar flux.

THE YAGLOM EQUATION AND SUBGRID-SCALE FLUX MODELLING

The DNS data are obtained using a pseudospectral method which discretizes space with Chebychev polynomials in the z direction and with Fourier modes in the x and y directions. Time integration is performed with a fourth order Runge-Kutta scheme for the nonlinear terms and a second order accurate Crank-Nicholson scheme for the linear ones. The simulation domain is a rectangular box of size $8 \times 8 \times 1$ along x, y, z respectively, where the x and y directions are parallel to the horizontal plates and the z-axis points in the direction opposite to that of gravity acceleration. Periodic boundary conditions are imposed at the lateral sidewalls whereas isothermal and noslip boundary conditions are used on the top and bottom plate, see figure 1 for an instantaneous view of the thermal field. The DNS is performed at $Pr = 0.7$ and $Ra = 1.7 \cdot 10^5$ using 128^2 (horizontal)$\times$129 (vertical) modes and polynomials, which yields a spatial resolution large enough to solve the smallest length scale of the problem.

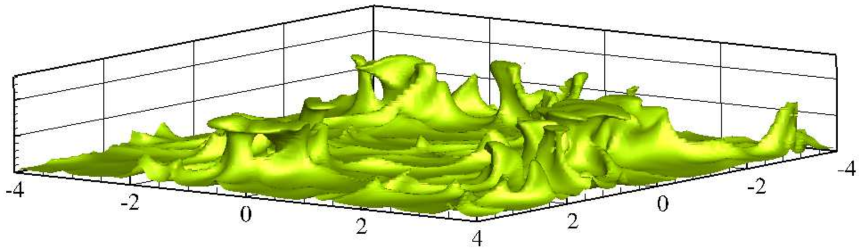

Figure 1. Direct Numerical Simulation of turbulent Rayleigh-Bénard convection in a laterally unbounded domain confined between two horizontal parallel walls. Isosurface of hot temperature, $\theta = 0.25$.

In LES, the large, energy carrying structures are directly solved while the small scales are modelled. In this scenario, a compound description in the physical/scale space is required for the correct understanding and modelling of the physics of the small unresolved thermal scales. Appropriate candidates to consider for a simultaneous description of turbulent dynamics in physical and scale space are the two-points statistical observables, such as the second-order structure function for the temperature $\langle \delta\theta^2 \rangle = \langle \delta\theta\delta\theta \rangle$, where $\delta\theta = \theta(x_i + r_i) - \theta(x_i)$ denotes the fluctuating increment of temperature θ between points x_i and $x_i' = x_i + r_i$. In the present flow, $\langle \delta\theta^2 \rangle$ depends both on the separation vector r_i and on the position in the wall-normal direction, i.e. $Z_c = (z' + z)/2$. The balance equation of $\langle \delta\theta^2 \rangle$ is the so-called Yaglom equation here generalized to inhomogeneous flows,

$$\frac{\partial \langle w^* \delta\theta^2 \rangle}{\partial Z_c} + 2 \langle w^* \delta\theta \rangle \frac{\partial \delta \langle \Theta \rangle}{\partial Z_c} - \frac{1}{2\sqrt{PrRa}} \frac{\partial^2 \langle \delta\theta^2 \rangle}{\partial Z_c^2} + \frac{\partial \langle \delta\theta^2 \delta u_i \rangle}{\partial r_i}$$
$$- \frac{2}{\sqrt{PrRa}} \frac{\partial^2 \langle \delta\theta^2 \rangle}{\partial r_j \partial r_j} + 2 \langle \delta w \delta\theta \rangle \left(\frac{\partial \langle \Theta \rangle}{\partial Z_c} \right)^* + 4 \langle \chi^* \rangle = 0, \tag{1}$$

where Θ is the total temperature field and $\langle \chi \rangle$ is the average rate of thermal dissipation. Equation (1) can be rewritten in a conservative form as

$$\frac{\partial \mathbf{\Phi}_r}{\partial r_i} + \frac{\partial \mathbf{\Phi}_c}{\partial Z_c} = \xi \tag{2}$$

allowing us to recognize two distinct fluxes of thermal energy occuring in such a flow. A flux in the space of scales, $\mathbf{\Phi}_r = \langle \delta\theta^2 \delta u_i \rangle - (2/\sqrt{PrRa}) \partial \langle \delta\theta^2 \rangle / \partial r_i$, and in physical space, $\mathbf{\Phi}_c = \langle w^* \delta\theta^2 \rangle - 1/(2\sqrt{PrRa}) \partial \langle \delta\theta^2 \rangle / \partial Z_c$. These two fluxes balance with a source term $\xi = -2 \langle w^* \delta\theta \rangle \partial \delta \langle \Theta \rangle / \partial Z_c - 2 \langle \delta w \delta\theta \rangle (\partial \langle \Theta \rangle / \partial Z_c)^* - 4 \langle \chi^* \rangle$. From the LES point of view, the most interesting term to study is $\mathbf{\Phi}_r$ since it represents for a given scale such as could be for the filter length, the net exchange of thermal energy between larger and smaller scales. Preliminary analysis (here not shown for brevity) highlight that this term exhibits a double features of forward and reverse transfer of thermal energy. In particular, in the near-wall region a strong reverse transfer from small to large scales is observed. Accordingly, we propose to model the subgrid scalar flux as

$$\tau_{i\theta} = C_\Delta \delta u_i \delta\theta - (C_S \Delta)^2 |\bar{S}| \frac{\partial \bar{\theta}}{\partial x_i} \tag{3}$$

where C_Δ is a free coefficient to be determined *a priori* or computed dynamically. Accordingly to the model for the subgrid stress tensor proposed in [3], the present model for the subgrid scalar flux (3) should be able to capture the double features of sourcing and draining of the small thermal scales of wall flows. The detailed study of the Yaglom equation, in particular of the transfer term in the space of scales, $\mathbf{\Phi}_r$, and the *a posteriori* analysis of an LES simulation performed with model (3) for the scalar field will be presented at the conference.

References

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