TERTIARY PATTERNS IN INCLINED LAYER CONVECTION

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Abstract Convection in an inclined layer generates various types of spatio-temporal patterns due to interaction of buoyancy and shear. At small angles of incline, the secondary instability of the uniform base state occurs in the form of buoyancy dominated longitudinal rolls. Above a critical angle of incline marking a co-dimension 2 point, shear driven transverse roll instabilities take over as the secondary instabilities. Computing the location of the co-dimension 2 point for varying thermal driving and inclination angle and determining all secondary bifurcations together with the resulting tertiary states allows to characterize the nonlinear phase diagram of inclined layer convection system. The semi-analytically computed phase diagram quantitatively matches experimental observations by Daniels *et al.* [1]. Close to the co-dimension 2 point, a subcritical secondary bifurcation leading to bistability is identified. In the bistable region, heteroclinic cycles generate bursting behavior.

Many physical systems evolve to asymptotic states that exhibit spatial and temporal variations in their properties. Patterns driven by convection have been investigated extensively for the Rayleigh-Bénard system. In the RBC system, buoyancy driven patterns are associated with a sequence of bifurcations of the uniform base state. Therefore, methods that exploit the linear instability such as weakly nonlinear analysis leading to amplitude equations, can be employed to analyse them. In contrast, shear driven patterns as in plane Couette flow, occur even when the associated base state is linearly stable. Analysis of such subcritical patterns requires a fully nonlinear analysis and thus remains challenging. To investigate the formation of spatio-temporal patterns due to the interaction of mean shear and buoyancy, we focus on the inclined layer convection (ILC) system. In the Inclined Layer Convection (ILC) cell, the fluid layer is inclined to the horizontal plane and subject to a temperature gradient. Three relevant parameters characterize this system: the ratio of momentum to thermal diffusivity (Prandtl number, Pr), the ratio of buoyancy to viscous forces (Rayleigh number, R) and the angle of inclination (γ).

Primary bifurcations of the uniform base state in the ILC system gives rise to secondary instabilities in the form of convective roll patterns. At small inclinations, the tilt only slightly perturbes the RBC system and predominantly breaks the in-plane isotropy of the convection cell by selecting a shear direction. As a result the characteristic convection rolls observed as secondary patterns align with the preferred direction along the incline (longitudinal rolls). As the inclination angle is increased, the strength of the mean shear flow increases while the strength of forces due to buoyancy decreases. As a result, at large inclinations shear driven patterns take over as secondary patterns and rolls are now observed to align with the shear direction transverse to the direction of inclination.

We determine all secondary bifurcations of the ILC system, using an iterative semi-analytical method to carry out the full nonlinear stability analysis in a periodic convection cell [2]. Figure 1 shows the location of secondary bifurcations in a nonlinear phase diagram for different angles of incline γ and reduced Rayleigh number $\epsilon = (R/R_C - 1)$. The Prandtl number chosen here is $Pr = 1.07$, for which the co-dimension 2 point is located at $\gamma_2 = 77.8°$. Colored markers are experimental observations by Daniels *et al.* [1] of configurations where the secondary roll instabilities become unstable to produce different tertiary patterns. Correspondingly colored solid lines are obtained from nonlinear stability analysis and match the locations of the experimental data. Asymptotic patterns in the mid-plane temperature fluctuations, resulting from the evolution of the full nonlinear equations close to the secondary bifurcations are shown in the bottom panels.

At very small angles, the longitudinal rolls are stable over a large range of thermal driving and become unstable to subharmonic oscillations at $\epsilon \approx 1$. The resulting subharmonic tertiary pattern is shown in panel (**i**). This pattern resembles a pearl-necklace like pattern of bright (cold) spots in a background of wavy longitudinal rolls. Here, every second wavy roll has the same phase which reflects the subharmonic nature of the instability. At intermediate angles of incline ($20° \leq \gamma \leq \gamma_2$), the longitudinal rolls become unstable to long wavelength perturbations and produce undulatory patterns as shown in (**ii**). Close to the co-dimension 2 point, the transverse rolls become unstable to three types of instabilities as shown in the right panel of Figure 1.

The system configurations where we observe transverse bursts in experiments (black squares in figure) match thresholds calculated for the Knot type instability (black line). The tertiary pattern introduced by the Knot instability is shown in panel (**iii**). At higher thermal driving, this transverse bursting behavior ceases and is replaced with longitudinal bursting behavior (pink downward triangles in figure). The threshold calculated for the onset of subharmonic secondary instability (pink line) matches the transition region from transverse to longitudinal bursts. The patterns generated by this subharmonic

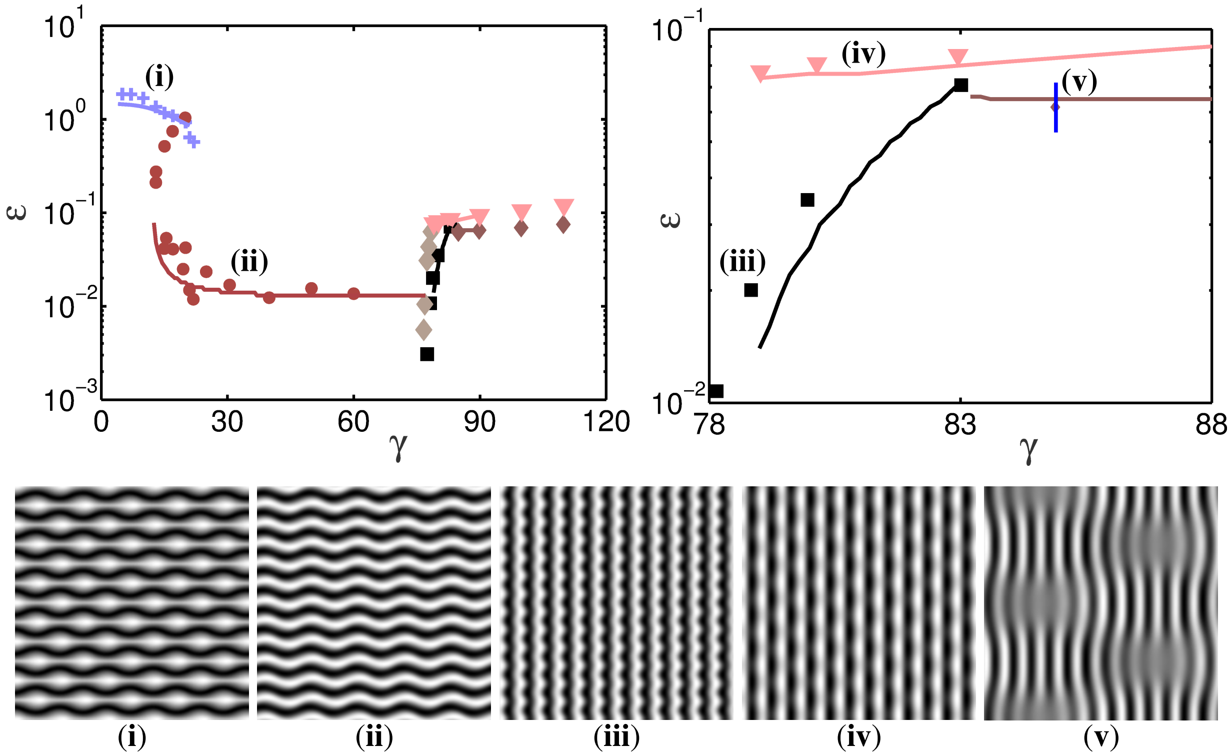

Figure 1. Left panel: The location of secondary bifurcations in nonlinear phase diagram for $Pr = 1.07$. Colored markers are from experimental observations of Daniels *et al.* [1]. Correspondingly colored solid lines are secondary bifurcations determined from nonlinear stability analysis. Right panel is the zoomed view close to the co-dimension 2 point. Here the blue line indicates the extent of bistable region at $\gamma = 84.9°$. Lower panels display mid-plane temperature fluctuations of tertiary patterns observed in the different regions of the nonlinear phase diagram.

varicose instability is shown in panel (**iv**). Here again the subharmonic nature of the instability results in the fact that the phase of every second transverse roll match. In the range ($\gamma \geq 83.2$), we also observe an oscillatory instability of the transverse rolls shown by the brown line in the right panel. The resulting tertiary pattern is shown in panel (**v**). Experiments close to these system configurations display switching diamond pane behavior (brown diamond).

The nonlinear stability analysis indicates that all secondary bifurcations are super-critical except for the oscillatory instability of the transverse rolls. To confirm this, we fix the angle of incline at $\gamma = 84.9°$ and vary the thermal driving ϵ. The critical value of instability of transverse rolls is $\epsilon = 0.065$. But we observe oscillatory roll patterns for values of thermal driving over ($\epsilon \geq 0.053$), denoted by the blue line in right panel of Figure 1. Thus, we observe bistability of transverse roll and oscillatory roll patterns in the range ($0.053 \leq \epsilon \leq 0.065$).

References

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