ENERGY EXCHANGES AND TIME ASYMMETRY IN 3D TURBULENT FLOWS

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Abstract In 3D turbulent flows, the direct cascade of energy, characterized by a flux through scales, ε, is a strong source of irreversibility. This irreversibility manifests itself in the asymmetry between negative and positive kinetic energy differences along particle trajectories. In particular, it is observed that the odd moments of the power of the forces acting on a fluid particle, $p \equiv \mathbf{a} \cdot \mathbf{V}$ are negative, and that $-\langle p^3 \rangle / \varepsilon^3 \propto R_\lambda^2$. This property rests on subtle correlations between acceleration and velocity. I will discuss two representations of the acceleration $\mathbf{a}$, which shed light on the irreversibility of the flow.

INTRODUCTION

The study of the motion of individual fluid particles transported by high Reynolds numbers flows raises significant questions on the physics of turbulence. Here, we focus on the motion of individual fluid particles. Contrary to the case of eulerian properties, the lagrangian structure functions: $D_n(\tau) = \langle (\mathbf{V}(\tau) - \mathbf{V}(0))^n \rangle$, where the average $\langle ... \rangle$ refers either to an ensemble average, or in the case of stationary flows, to a time-average along a trajectory, are insensitive to a time reveral of the motion of particles, and therefore, they can provide at best limited information on the dynamics [1].
Recent experiments and numerical simulations revealed that energy differences along a fluid trajectory, $W(\tau) = \frac{1}{2}(\mathbf{V}^2(\tau) - \mathbf{V}^2(0))$, are sensitive to the lack of time symmetry. Qualitatively, large negative values of W are more probable than large positive values of W, an asymmetry which gives rise to negative odd moments of W. In the short τ limit, the n^{th} moments of W reduce to the n^{th} moments of power, $p \equiv \mathbf{V} \cdot \mathbf{a}$, multiplied by τ^n. Experimental and numerical results in particular reveal that $-\langle (p/\varepsilon)^3 \rangle \propto R_\lambda^2$, over a large range of Reynolds numbers $100 \leq R_\lambda \leq 900$.
To better understand this result, we introduce here two decompositions of the acceleration. The first decomposition is based on identifying the forces acting on a fluid. In a forced flow, acceleration is simply expressed as:

$$\mathbf{a} = -\nabla P/\rho + \mathbf{D} + \mathbf{f} \tag{1}$$

where P is pressure, ρ is the fluid density, which we can take without any loss of generality to be equal to 1, $\mathbf{D}$ the dissipation, which is due to viscosity, and $\mathbf{f}$ is the large-scale forcing. It is known that the dominant term in the decomposition (1) is due to pressure. We demonstrate that the role of pressure is more subtle, and that paradoxically, the term $-\mathbf{V} \cdot \nabla P$ has a *positive* third moment. Further analysis reveals a surprising role of pressure, in accelerating fast particles.
Another way to represent acceleration consists in writing $\mathbf{a}$ as a sum of the local part, plus a convective part: $\mathbf{a} \equiv \mathbf{a}_L + \mathbf{a}_C$, with $\mathbf{a}_L = \partial_t \mathbf{u}$ and $\mathbf{a}_C = (\mathbf{u} \cdot \nabla)\mathbf{u}$. The term $\mathbf{a}_L$ is due to the eulerian time-dependence of the velocity field, whereas the term $\mathbf{a}_C$ corresponds to the acceleration of a particle in a frozen velocity field $\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_0(\mathbf{x})$. We show that integrating the third moment of the velocity difference, $\langle W^3(\tau) \rangle$ is also negative in the case of a frozen velocity field.

RESULTS

The results presented here are based on direct numerical simulations of the Navier-Stokes equations, by using pseudo-spectral codes, run at ENS Lyon, with Reynolds numbers in the range: $115 \leq R_\lambda \leq 300$ [5, 3]. In addition, we have used data obtained from the Johns Hopkins Turbulent Database, at $R_\lambda = 430$.

Role of pressure

For all the Reynolds number considered here, the pressure gradient contributes to more than $\sim 85\%$ of the variance of p [3], which is consistent with the observation of [4]. On the other side, the asymmetry in the fluctuations of p is not captured by the fluctuations of $-\mathbf{V} \cdot \nabla P$. In fact, whereas the skewness of p is found to be of the order ~ -0.6, the skewness of the fluctuations of $-(\mathbf{V} \cdot \nabla)P$ is found to be of order 0.1. The difference in sign between the third moment of p and the pressure term $-\mathbf{V}\nabla P$ suggests that pressure tends to contribute more to large energy gain than to energy losses, which seems a bit counterintuitive. In fact, the third moment of $p = -\mathbf{V} \cdot \mathbf{a}$, where $\mathbf{a}$ is decomposed as in (1), is mostly due to the cross-product terms $3\langle (-\mathbf{V} \cdot \nabla P)^2 (\mathbf{V} \cdot \mathbf{D}) \rangle$ and $3\langle (-\mathbf{V} \cdot \nabla P)^2 (\mathbf{V} \cdot \mathbf{f}) \rangle$.
Generally, it is known that pressure does not play any role in the energy balance in a homogeneous flow: $\langle \mathbf{V} \cdot \nabla P \rangle = 0$. Although this condition imposes that pressure does not create or dissipate any energy, pressure may still redistribute

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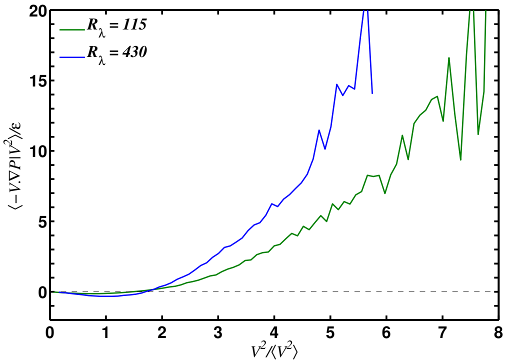

Figure 1. The conditional average of $-(\mathbf{V} \cdot \nabla)$ on V^2 shows that particles with a large velocity tend to be accelerated by pressure forces.

energy between particles. Fig. 1 shows the value of $\langle -(\mathbf{V} \cdot \nabla) P | \mathbf{V}^2 \rangle$, and demonstrates that pressure tends to increase the velocity of fast particles, at the expanse of slower particles. This surprising result may point to a contribution of pressure in the problem of singularities in the Navier-Stokes equations [2], to be further understood.

Asymmetry of energy in frozen turbulent flows

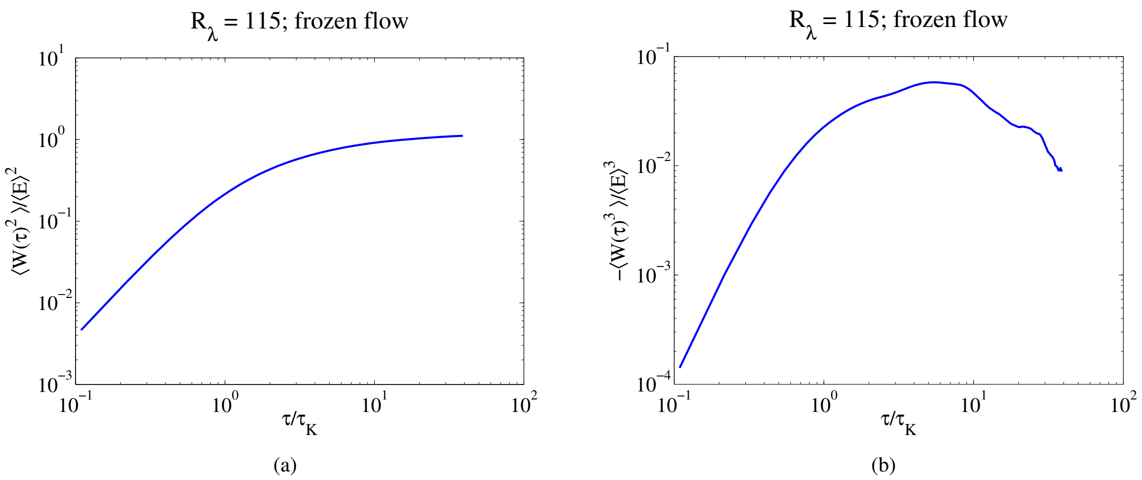

Figure 2. The moments $\langle W^2(\tau) \rangle / \langle E \rangle^2$ (left) $-\langle W^3(\tau) \rangle / \langle E \rangle^3$ (right) as a function of τ, in the case of a frozen flow at $R_\lambda = 115$

Fig. 2 shows the second (left) and third (right) moments of W, obtained from a frozen turbulent flow at $R_\lambda = 115$. The third moment of W is negative, suggesting that some of the properties uncovered in [5] may be analyzed in terms of spatial properties of turbulent flows.

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