STOCHASTIC SUBGRID ACCELERATION MODEL FOR INERTIAL PARTICLES IN LES OF A HIGH REYNOLDS NUMBER FLOW

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Abstract In the context of Large Eddy Simulation (LES) of turbulent flow laden by solid particles, we propose a modelling of the the interaction of a particle with the unresolved scales of the flow. We consider both particles much smaller and larger than the Kolmogorov length scale. The small scales of high Reynolds number flow are characterized by strong velocity gradients. To account for those gradients, and specifically the turbulent time-scales shorter than the Stokes time, we decompose the particle acceleration in its resolved and residual parts. In the latter, the interactions with the inertial range of the turbulent flow are simulated by a stochastic process evolving along the particle trajectory. For the case of the small particles, we introduced two processes, one for its norm, and another for its direction. Results showed that by introducing the stochastic model for the particle residual acceleration, the particle acceleration statistics from DNS is predicted fairly well. For the particles bigger than the Kolmogorov scale, we propose another stochastic model. We derived a fluctuating drag, simulated by lognormal process. This model gives stretched tails in the particle acceleration distribution invariant with the density and the size of particle as observed experimentally.

Amongst fundamental questions that arise in LES of turbulent flows laden with a dispersed phase, one is how to simulate the response of a single inertial particle to turbulent frequencies that are not resolved in the simulation. The motivation is this: at large Reynolds number, the flow structure at small unresolved scales may be strongly intermittent. Consequently, the unresolved gradients of the fluid velocity, and the kinetic energy dissipation rate, may fluctuate widely at high frequencies and at larges amplitudes. Therefore at a high Reynolds number, it is expected that the inertial particle along its path is responsive to such flow fluctuations at subgrid scales. In the LES framework, we derive the decomposition of the instantaneous acceleration of the solid (or liquid) particle on two parts $\frac{d\mathbf{u}_p}{dt} = \overline{\left(\frac{d\mathbf{u}_p}{dt}\right)} + \left.\frac{d\mathbf{u}_p}{dt}\right|_{\varepsilon}$, where on the right hand side, the first part is given by the steady-drag force in the resolved velocity field, and the second term denotes for the particle acceleration conditionally averaged on the instantaneous dissipation rate ε along the particle path. The latter is modeled from the presumed statistical properties of ε with the locally defined parameters obtained from the resolved field. In our work, we propose and assess such a model for inertial particles flying in a LES box turbulence at high Reynolds number. Two different cases were considered: (i) the particle diameter is greater than the Kolmogorov scale, $d_p > \eta$; (ii) the particle diameter is less than the Kolmogorov scale, $d_p < \eta$.

To begin with the case (i), we suggest that when $d_p > \eta$, the drag force is controlled not only by laminar viscous effects, but also by the instanteneous rate of dissipation of turbulent kinetic energy ε_{d_p} averaged over $\ell_{turb} \sim d_p$. To this end, we rewrite the particle response time as $\tau_{p,t} = \frac{\rho_p}{\rho_f}\frac{d_p^2}{\nu + \nu_{p,t}}$ Scaling of the turbulent viscosity is obtained in terms of the Kolmogorov-Oboukhov 62 theory[2] $\nu_{p,t} \sim \langle (\Delta_{d_p} u) | \varepsilon_{d_p} = \varepsilon \rangle = \varepsilon^{1/3} d_p^{4/3}$, we obtain the following equation for the particle motion: $\frac{d\mathbf{u}_p}{dt} = \frac{\overline{\mathbf{u}}_f - \mathbf{u}_p}{\tau_p} + \frac{\rho_f}{\rho_p}\frac{\varepsilon^{1/3}}{d_p^{2/3}}(\overline{\mathbf{u}}_f - \mathbf{u}_p)$. Here the over line denotes for the filtered velocity at the particle location, and $\varepsilon^{1/3}$ is a sample-space variable. The filter width is considered to be greater than the particle diameter, $\Delta > d_p$. To estimate the value of $\varepsilon^{1/3}$, we apply the Ito transformation to the Pope & Chen stochastic equation for ε [3]. As an illustration, the computed PDF of the particle acceleration corresponding to five particle diameters: $d_p/\eta = 0.3$, 0.6, 1, 1.5 and 2 and with density ratio $\rho_p/\rho = 900$ are given in Fig. 1a. The eddy-viscosity ν_{turb} is represented by the Smagorinsky model; the Reynolds number based on the Taylor scale is $R_\lambda = 400$, and the mesh consists in 64^3 nodes. For comparison, PDFs obtained from simulations using the classical Stokes drag law are also show. The both approaches are compared with measurements [4]. It is seen that the result obtained with the standard Stokes law present a less stretched tails than the experiment and becomes narrower with increasing the particle diameter. In the experiment of Quershi et al. (2008) it has been observed that the shape of the PDF remains unchanged with modification of the particle size and density. We observe, in Fig. 2 that our model correctly predict the behavior observed experimentally.

For the case (ii), concerning small inertial particles, we propose to model the particle acceleration resulting from the subgrid scale of the flow by the product of two independent stochastic processes: $\left.\frac{d\mathbf{u}_p}{dt}\right|_{\varepsilon} = a'_p(t)\mathbf{e}_p$. a'_p is the random modulus of the particle acceleration at subgrid scales, and $\mathbf{e}_p$ is its random unit vector of orientation. The stochastic process for a'_p is defined from the random variable ε: $a'_p = \sqrt{\varepsilon/\tau_p}$. Here again, we applied the Ito transformation to the Pope & Chen stochastic equation [3]. The second stochastic process for the unit vector of the orientation $\mathbf{e}_p(t)$ is given by a Brownian motion on the unit sphere, assuming that orientation components stay correlated over the Kolmogorov's timescale τ_η. The variance of the particle acceleration computed for five Stokes numbers: ($St = 0.16$, 0.6, 1, 2) are given in Fig. 1b. The Reynolds number, the mesh resolution and eddy viscosity model are similar to the case (i). The variance are compared with LES using the Standard Stokes law for the evolution of the particles at three resolutions (64^3, 96^3, 128^3), and DNS form Toschi et al. [1]. The decrease of variance with the increase of the Stokes number observed from the DNS is well reproduced by the LES coupled with the proposed stochastic model, whereas the standard LES reproduces much less accurately this effect of the Stokes number even with a much finer spatial resolution.

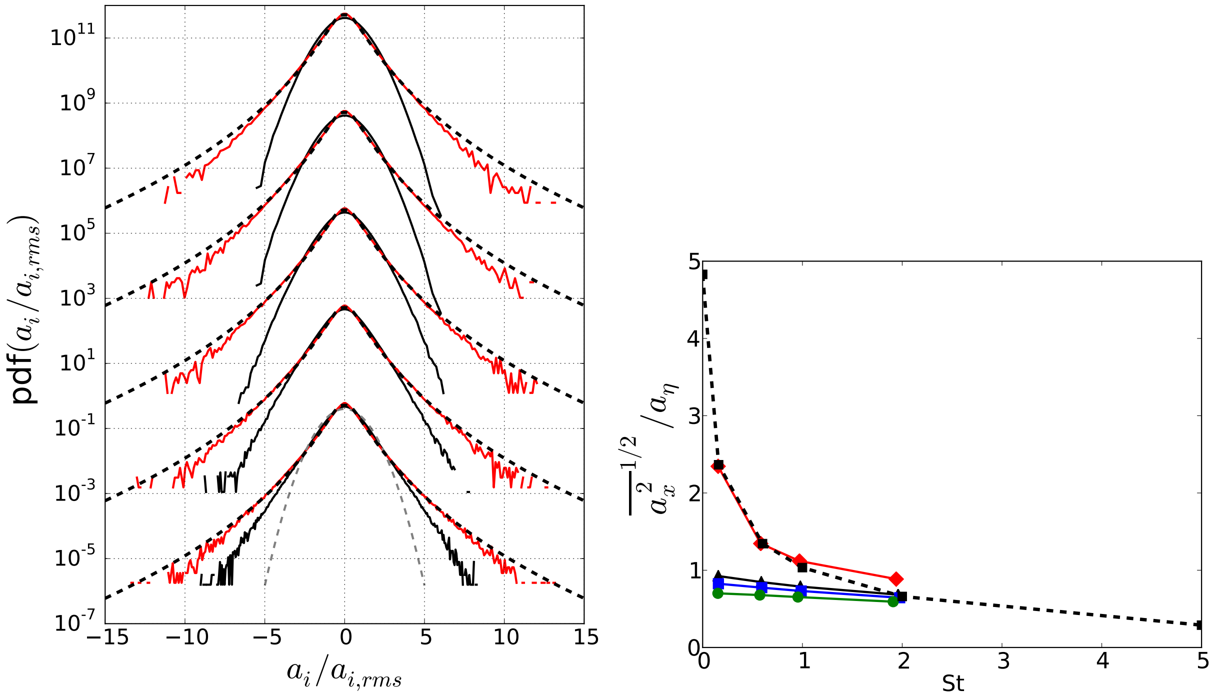

Figure 1. (left) PDF of the particle acceleration normalized by the variance for 5 particle diameters (equal to 0.3, 0.6, 1, 1.5 and 2 times the Kolmogorov length from bottom to top. LES with standard Stokes drag (black lines), LES with the stochastic model (red lines), the normal distribution (grey dashed line) and the experimental fit from Qureshi et al. [4] (black dashed lines). (right) Evolution of variance of the particle acceleration with the Stokes number. LES with the stochastic model and with 64^3 mesh: red; LES with standard Stokes drag for three resolution (64^3: green, 96^3: blue, 128^3: black), and DNS form Toschi et al.: dashed line.

References

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