ENERGY TRANSFER IN ROTATING STRATIFIED TURBULENT FLOWS

Raffaele Marino[1,2], Annick Pouquet[1,3], Duane L. Rosenberg[4] & Pablo D. Mininni[5]
[1] *National Center for Atmospheric Research, P.O. Box 3000, Boulder, CO 80307, USA*
[2] *Institute for Chemical-Physical Processes - IPCF/CNR, Rende (CS), 87036, Italy*
[3] *Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, CO 80309, USA*
[4] *National Center for Computational Sciences, Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, TN 37831, USA*
[5] *Universidad de Buenos Aires & IFIBA, CONICET, Ciudad Universitaria, 1428 Buenos Aires, ARGENTINA*

Abstract When including rotation and stratification non-linear interactions in turbulent flows occur both at small scales, with turbulent mixing and dissipation, as well as at large scales where accumulation of energy can take place through an inverse cascade. We present a characterization of the energy transfer across the scales in rotating and/or stratified turbulent flows showing how the spectra and fluxes in the Fourier space depend on the relative strength between rotation and stratification.

INTRODUCTION

Geophysical flows are turbulent, with complex systems of eddies interacting non-linearly, making such flows largely unpredictable. They also support strong waves, due to the combined effect of rotation and stratification. Under large-scale quasi-geostrophic regime (a balance established between gravity, rotation, and pressure gradients), the motions are quasi-two-dimensional and the flow of energy is thought to be predominantly to the large scales. However, this poses several questions: what is the distribution of energy among modes (the Fourier spectrum)? What is the role of anisotropy, and under what conditions does the system self-organize and develop large-scale structures? Direct numerical simulations (DNS) have shown that rotating flows in finite domains can develop inverse cascades of energy and more recently we demonstrated that, in the *Boussinesq* framework, rotating stratified flows support a bi-directional energy cascade with a constant energy flux to both large and small scales [1]. In the case of purely stratified flows evidence for the existence of an inverse cascade is less clear, and conclusions from simulations are sometimes contradictory. We present here results from high-resolution DNS of rotating and/or stratified flows to describe characteristics of the energy cascade in rotating stratified flows and to point out the existence of a partial inverse energy transfer even in *Boussinesq* flows without rotation.

DIRECT NUMERICAL SIMULATIONS

We use the code GHOST (*Geophysical High Order Suite for Turbulence*), a pseudo-spectral DNS framework [2, 3], to solve the *Boussinesq* equations for an incompressible, rotating and stratified fluid:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\nabla^2\mathbf{u} = -\nabla p - N\theta\hat{z} - f\hat{z}\times\mathbf{u} + \mathbf{F}, \tag{1}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u}\cdot\nabla\theta - \kappa\nabla^2\theta = N\mathbf{u}\cdot\hat{z}, \tag{2}$$

$$\nabla\cdot\mathbf{u} = 0. \tag{3}$$

Here, $\mathbf{u}$ is the velocity, θ the (potential) temperature fluctuations, and p the pressure normalized by a unit mass density. The pressure is obtained self-consistently from the incompressibility condition. We consider a Prandtl number $Pr = \nu/\kappa = 1$, with ν the kinematic viscosity and κ the temperature diffusivity. In Eqs. (1) and (2), N is the Brunt-Väisälä frequency, and $f = 2\Omega$ with Ω the rotation frequency. Runs were performed on grids of $n^3 = 512^3$ and $n^3 = 1024^3$ points with an external mechanical force $\mathbf{F}$ (isotropic and randomly generated) applied in a shell of modes with wave numbers k_F, respectively with $k_F = [22, 23]$ and $k_F = [40, 41]$. These choices for the forcing wave number allow for sufficient scale separation to study an inverse energy transfer towards the large scales.

RESULTS

We have found that in rotating stratified turbulent (RST) flows the inverse cascade of kinetic energy is more efficient than in the equivalent purely rotating case when a moderate level of stratification is introduced [4]. In particular, our results show that the growth rate of kinetic energy ($\sigma = dE_V/dt$) in a forced RST flow is faster for $1/2 < N/f < 2$ (see Fig. 1). This is also the range in which the triadic wave resonances disappear [5]. For values of $N/f > 2$ the efficiency of the inverse cascade decreases monotonically until the limit of purely stratified flows where a negligible

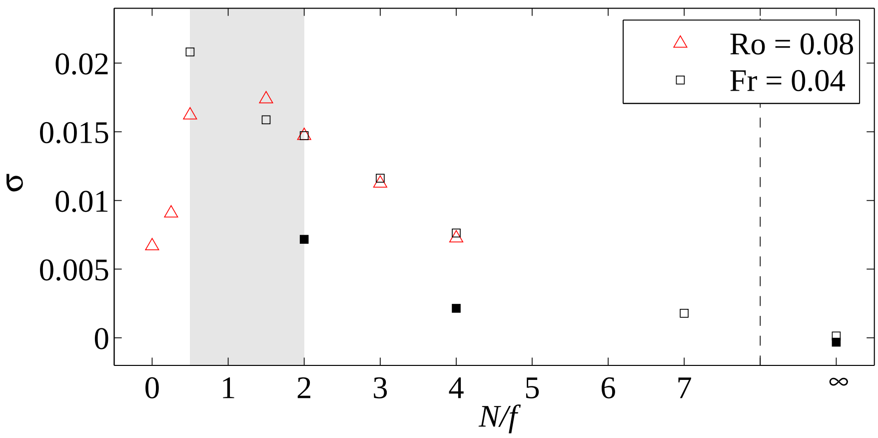

Figure 1. *Marino et al., 2013*. Growth rate of the kinetic energy as a function of N/f for a variety runs. Empty symbols correspond to 512^3 runs, the filled to 1024^3 runs. Same values of N/f are obtained for different combinations of the *Froude* (*Fr*) and *Rossby* (*Ro*) numbers. The gray band indicates the range $1/2 < N/f < 2$, where the energy grows faster.

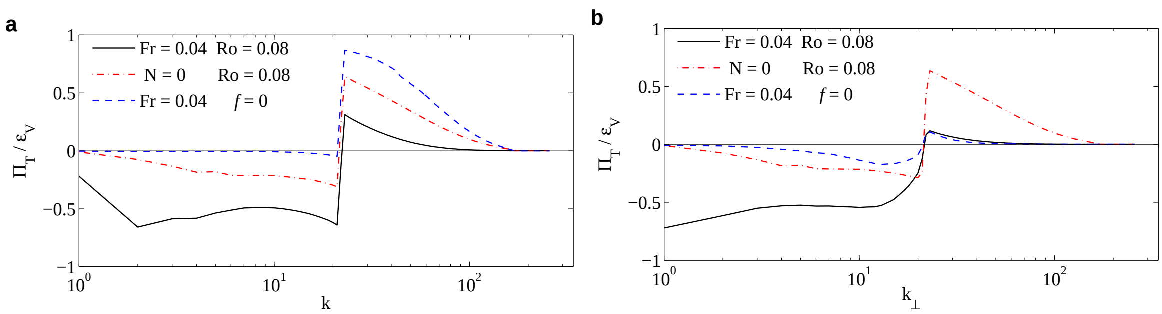

Figure 2. *Marino et al., 2014*. (a) Isotropic and (b) perpendicular total energy fluxes normalized by the energy input $\epsilon_V = \langle \mathbf{u} \cdot \mathbf{F} \rangle$ in a run with $N/f = 2$ (solid, black line), a run with pure rotation (dash-dotted, red line), and a run with pure stratification (dashed, blue line).

isotropic large scale transfer can be diagnosed with a nearly zero isotropic flux for $k < k_F$ (Fig. 2a). However, in the purely stratified case we find that when sufficient scale separation is allowed between the forcing scale and the domain size, the reduced kinetic energy spectrum $E_V(k_\perp)$ displays at large scales a power-law behavior in the perpendicular direction compatible with $\sim k_\perp^{5/3}$. This spectrum is the result of the combined inverse transfer (negative flux in Fig. 2b, blue line) in the perpendicular direction ($k_\perp$) and a direct cascade manifested by a constant positive flux of energy in the parallel direction at the largest scales ($k_{||}$) [6]. A parametric study at higher resolution (up to 2048^3) for $2 < N/f < 10.5$ and $1 < \mathcal{R}_\mathcal{B} = Re * Fr^2 < 300$ is in progress to ascertain what determines the ratio for the inverse to the direct transfer of energy when a bi-directional energy cascade is observed in DNS of rotating stratified flows forced at intermediate scales ($k_F = [10, 11]$), as in the case of the *abyssal Southern Ocean* at mid latitudes [7].

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