BENDING DYNAMICS OF SEMI-FLEXIBLE PARTICLES IN TURBULENT FLOWS

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Abstract We study the Lagrangian dynamics of semi-flexible particles in laminar as well as in homogeneous and isotropic turbulent flows by means of analytically solvable stochastic models and direct numerical simulations. The statistics of the bending angle is qualitatively different in laminar and turbulent flows and exhibits a strong dependence on the topology of the velocity field. In particular, in two-dimensional turbulence, particles are either found in a fully extended or in a fully folded configuration; in three dimensions, the predominant configuration is the fully extended one.

INTRODUCTION

The study of hydrodynamic turbulence and turbulent transport has received considerable impulse from the development of experimental, theoretical, and numerical Lagrangian techniques [1, 4, 2, 3]. The translational dynamics of tracer and inertial particles is related to the mixing properties of turbulent flows [5] with applications in geophysics (atmospheric pollution, rain formation) [6], astrophysics (dynamo effect, planet formation) [7, 8], and chemical engineering [9]. Over the last decade, attention has extended to the Lagrangian dynamics of particles possessing additional degrees of freedom, such as elastic polymers and nonspherical solid particles (see, e.g., Refs. [10, 11] and references therein). In particular, the examination of the extensional dynamics of polymers provides information on the coil-stretch transition and turbulent drag reduction [12]; recent studies of the dynamics of non-spherical particles in isotropic turbulence have investigated their alignment and rotation statistics of such particles [13, 14, 15, 16, 17]. We consider particles that possess yet another degree of freedom, namely semi-flexible particles that can bend under the action of a non-uniform velocity field. We model a semi-flexible particle as a trumbbell, i.e., as three beads connected by two rigid links [18] (see Fig. 1); the stiffness of the trumbbell is controlled by an elastic hinge, which prevents the particle from bending. The trumbbell model (also known as trimer or three-bead-two-rod model) qualitatively captures the viscoelastic properties of suspensions of segmentally flexible macromolecules [18, 19, 20, 21, 22, 23, 24]. This model is also important in statistical mechanics as the prototypical system for showing that the infinite-stiffness limit of elastic bonds is singular [18, 25, 26, 27, 28]. Moreover, its active variant, the trumbbell swimmer, describes the swimming motion of certain biological microorganisms [29, 30] (see also Ref. [31]).

STATISTICS OF THE BENDING ANGLE

We study the bending dynamics of trumbbells, in both laminar and fully turbulent flows, via analytical calculations and detailed numerical simulations. We show that the statistics of the bending angle depends strongly on the topology of the flow.

In particular, in the two-dimensional Batchelor–Kraichnan flow (a Gaussian random flow with zero correlation time and power-law spatial correlations), the configurations in which the rods are parallel or antiparallel are most probable; as the amplitude of the velocity gradient increases, these probabilities become sharper, with the parallel configuration becoming the most likely for very strong turbulence (see Fig. 2). By contrast, in the three dimensional Batchelor–Kraichnan flow, the antiparallel configuration always is the most probable one (see Fig. 2). The above analytical results are confirmed by direct numerical simulations of homogeneous and isotropic turbulence.

This fact suggests that the rheology of semiflexible macromolecules also exhibits a strong dependence on the turbulent character of the flow and on its spatial dimension. In particular, in 2D turbulence the feedback on the flow may be more intense than in 3D, as trumbbells oscillate between the folded and extended configurations. The recent advances in Lagrangian experimental techniques allow extensive laboratory studies of the deformation of macromolecules in turbulent

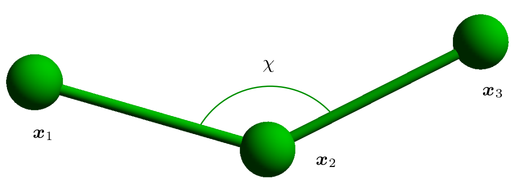

Figure 1. The trumbbell model.

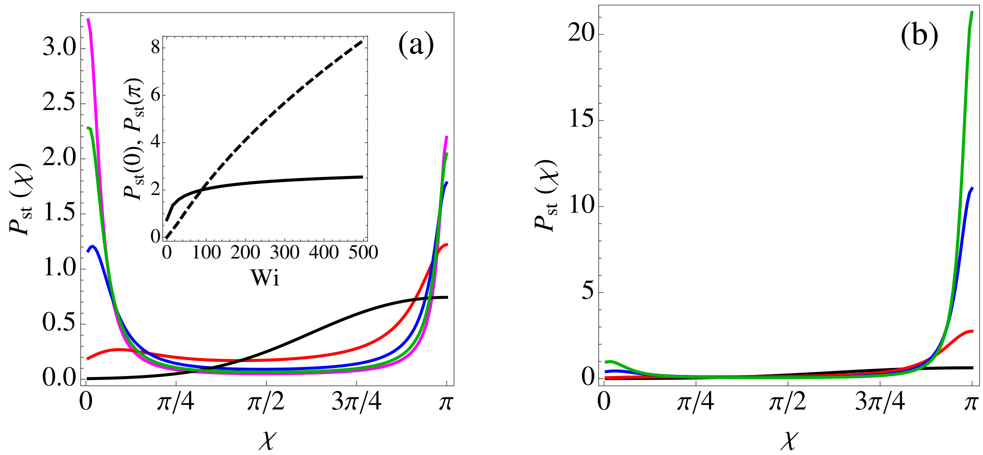

Figure 2. Stationary distribution function of χ for the Batchelor-Kraichnan flow with $Z = 1$ and $\mathrm{Wi} = 0$ (black), $\mathrm{Wi} = 10$ (red), $\mathrm{Wi} = 50$ (blue), $\mathrm{Wi} = 100$ (green), and $\mathrm{Wi} = 150$ (magenta) in (a) two and (b) three dimensions. The inset in (a) shows the values of $P_{\rm st}(\chi)$ at $\chi = 0$ (dashed line) and $\chi = \pi$ (solid line) as a function of Wi.

flows [4, 12]. We hope our work would lead to further experiments directed towards the study of the Lagrangian dynamics of semi-flexible macromolecules in turbulent flows and the non-Newtonian properties of turbulent suspensions of such particles, as well as theoretical investigations of models that incorporate effects such as hydrodynamic interactions and multiple relaxation modes.

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