REAL-SPACE MANIFESTATIONS OF BOTTLENECKS IN TURBULENCE SPECTRA

Uriel Frisch [1], Samriddhi Sankar Ray [2], G. Sahoo [3], Debarghya Banerjee [4] & Rahul Pandit [5]
[1]*Laboratoire Lagrange, OCA, UNS, CNRS, Nice, Cedex 4, France*
[2]*International Center for Theoretical Sciences, Tata Institute of Fundamental Research, Bangalore, India*
[3]*Department of Physics, University of Rome Tor Vergata, Rome, Italy*
[4]*Instuut-Lorentz, University Leiden, Leiden, The Netherlands*
[5] *Department of Physics, Indian Institute of Science, Bangalore, India*

Abstract An energy-spectrum bottleneck, a bump in the turbulence spectrum between the inertial and dissipation ranges, is shown to occur in the non-turbulent, one-dimensional, hyperviscous Burgers equation and found to be the Fourier-space signature of oscillations in the real-space velocity, which are explained by boundary-layer-expansion techniques. Pseudospectral simulations are used to show that such oscillations occur in velocity correlation functions in one- and three-dimensional hyperviscous hydrodynamical equations that display genuine turbulence.

INTRODUCTION

The energy spectrum $E(k)$ characterizes the statistical distribution of kinetic energy among the wavenumbers k in homogeneous, isotropic, fluid turbulence in three dimensions (3D). If k_I and k_d denote, respectively, the wave-vector magnitudes related to the inverses of the lengths L_I, at which energy is injected into the system, and η_d, where viscous dissipation becomes significant, then, in the inertial range $k_I \ll k \ll k_d$, this spectrum scales as $E(k) \propto k^{-n}$; the phenomenological theory (K41) of Kolmogorov [1], which does not account for intermittency [2], yields $n = 5/3$ for 3D fluid turbulence. In the far dissipation range $k \gg k_d$, this spectrum falls off exponentially (upto algebraic prefactors) [3]. For values of k that lie in between inertial and far-dissipation ranges, a plot of the *compensated* energy spectrum ($E(k)$ divided by its inertial-range form) versus k exhibits a gentle maximum that is called a *bottleneck* [4, 5]. Such bottlenecks have been seen in a variety of experiments [6, 7] and in direct numerical simulations (DNSs) of fluid turbulence [7, 8, 9, 10]. Phenomenological mechanisms have been suggested for the formation of bottlenecks (see, e.g., Refs. [4, 5]). A systematic theoretical study of the bottleneck phenomenon has been initiated in Ref. [11] by using the limit of high dissipativity α in hyperviscous hydrodynamical equations, which have a dissipation operator $\propto (-\nabla^2)^\alpha$. Hyperviscous dissipation, with moderate values of α, say between 2 and 4, is often used in DNSs in the hope of enhancing the inertial range of scales, but at the price of producing increasingly strong bottlenecks (see Ref. [11] and references therein).

RESULTS

In the first part of our study we develop a quantitative, analytical understanding of bottlenecks, for moderate values of α, in the following hyperviscous generalization of the one-dimensional (1D) Burgers equation:

$$\partial_t u + u\partial_x u = -\nu_\alpha k_r^{-2\alpha}(-\partial_x^2)^\alpha u + f(x,t); \tag{1}$$

here $u(x,t)$ is the velocity at the point x and time t, $\nu_\alpha > 0$ the hyperviscosity, k_r a reference wavenumber, and f the driving force. It is well known that the ordinary ($\alpha = 1$) Burgers equation, with $f = 0$, is integrable [12]; it is also easy to show that its energy spectrum has no bottleneck. By contrast, we show that, for any integer $\alpha > 1$, the solution to the hyperviscous Burgers equation (1), in the limit of small ν_α, displays an energy-spectrum bottleneck; for this it is crucial to examine the solution in real space, where we can use boundary-layer-type analysis, in the vicinities of shocks, to uncover oscillations in the velocity profile. We obtain this result both for the unforced, hyperviscous Burgers equation and for its variant (DHB) with deterministic, time-independent, large-scale forcing. We validate our DHB solutions with a pseudospectral DNS. Note that these solutions are time-independent and not turbulent; however, the key qualitative feature of real-space oscillations in the velocity profile does carry over to oscillations in velocity correlation functions in one- and three-dimensional hyperviscous hydrodynamical equations that display genuine turbulence. We show this in the second part of our study by using DNS. This association of bottlenecks and oscillations in velocity correlation functions has not been made so far. It is akin to the association of peaks in the static structure factor $S(k)$, of a liquid in equilibrium, with damped oscillations in the radial distribution function $g(r)$ [13].

References

[1] A.N. Kolmogorov, *Dokl. Akad. Nauk. SSSR* **30**, 299âĂŞ303 (1941).
[2] U. Frisch, *Turbulence: the Legacy of A.N. Kolmogorov*, (Cambridge University Press, Cambridge, UK, 1995).
[3] J. von Neumann, Recent theories of turbulence (1949). In *Collected works (1949–1963)* **6**, 37–472, ed. A.H. Taub. Pergamon Press, New York, 1963; as first observed by von Neumann, this follows from the (conjectured) analyticity of the solution of the Navier–Stokes equation.

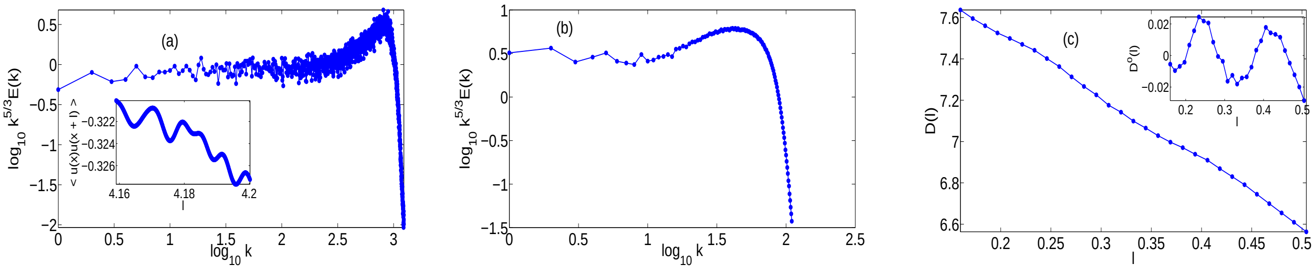

Figure 1. (a) Log-log plot of the compensated energy spectrum $k^{5/3}E(k)$ versus k for the stochastically forced hyperviscous Burgers (SHB) equation with $\alpha = 8$ and (inset) a plot of its correlation function $\langle u(x)u(x+l)\rangle$ showing oscillations of wavelength $\lambda_8^{\rm SHB}$, which is inversely related to the wavenumber of the bottleneck in the energy spectrum. (b) The compensated energy spectrum $k^{5/3}E(k)$ for the 3D HNS equation ($\alpha = 4$) with a bottleneck peak at wavenumber $K_{b,\alpha}^{\rm HNS} = 40$. (c) A plot of the correlation function $D(l)$ versus l for the 3D HNS equation; inset: oscillations in a plot of the function $D^o(l)$, which we obtain by subtracting the linear, decaying trend from $D(l)$.

[4] G. Falkovich, *Phys. Fluids*, **6**, 1411 (1994).

[5] D. Lohse and A. Müller-Groelling, *Phys. Rev. Lett.*, **74**, 1747 (1995).

[6] H.K. Pak, W.I. Goldburg, A. Sirivat, *Fluid Dynamics Research*, **8**, 19 (1991); Z.-S. She and E. Jackson, *Phys. Fluids A*, **5**, 1526 (1993); S.G. Saddoughi and S.V. Veeravalli, *J. Fluid Mech.*, **268**, 333 (1994).

[7] W. Dobler, N.E.L. Haugen, T.A. Yousef and A. Brandenburg, *Phys. Rev. E*, **68**, 026304 (2003).

[8] Z.-S. She, G. Doolen, R.H. Kraichnan, and S.A. Orszag, *Phys. Rev. Lett.*, **70**, 3251 (1993); P.K. Yeung and Y. Zhou, *Phys. Rev. E*, **56**, 1746 (1997); T. Gotoh, D. Fukayama, and T. Nakano, *Phys. Fluids*, **14**, 1065 (2002); M.K. Verma and D.A. Donzis, *J. Phys. A: Math. Theor.*, **40**, 4401 (2007); P.D. Mininni, A. Alexakis, and A. Pouquet, *Phys. Rev. E*, **77**, 036306 (2008).

[9] Y. Kaneda, *et al.*, *Phys. Fluids*, **15**, L21 (2003); T. Isihara, T. Gotoh, and Y. Kaneda *Annu. Rev. Fluid Mech.*, **41**, 165 (2009); S. Kurien, M.A. Taylor, and T. Matsumoto, *Phys. Rev. E*, **69**, 066313 (2004).

[10] D.A. Donzis and K.R. Sreenivasan *J. Fluid Mech.*, **657**, 171 (2010); DOI: 10.1017/S0022112010001400, suggest that bottlenecks might go away as the Reynolds number goes to infinity.

[11] U. Frisch, *et al.*, *Phys. Rev. Lett.*, **101**, 144501 (2008).

[12] J. Bec and K. Khanin, *Phys. Rep.*, **447**, 1 - 66 (2007).

[13] See, e.g., J.-P. Hansen and I.R. McDonald, *Theory of Simple Liquids*, (Academic Press, London, 2006); J.L. Yarnell, M.J. Katz, R.G. Wenzel, and S.H. Koenig, Phys. Rev. A, **7**, 2130 (1973).