A PRIORI AND A POSTERIORI ANALYSIS OF THE HYBRID TWO-LEVEL LARGE-EDDY SIMULATION METHOD FOR HIGH REYNOLDS NUMBER COMPLEX FLOWS

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INTRODUCTION

Turbulent flows at high Reynolds number (Re) occur in several engineering and natural systems. Such flows exhibit presence of features such as transition and non-equilibrium behavior, separation/reattachment, skewed boundary-layers etc. An accurate and efficient prediction of such flow features is necessary from design perspective. Numerical investigation of such flows becomes challenging for conventional numerical methods due to their multi-scale nature. Large-eddy simulation (LES) is considered to be a promising approach to simulate such flows [2]. However, LES on its own suffers from several limitations due to the employed spatial filtering and eddy viscosity concepts. These limitations have led to development of techniques such as hybrid RANS-LES [5] and multi-scale [6, 7] formulations. The two-level simulation (TLS) [7] and its extension, the hybrid two-level large-eddy simulation (TLS-LES) [4, 12] models are multi-scale methods, which alleviate some of the challenges faced by LES. In this study, we perform a priori and a posteriori analysis of the assumptions and predictions of the hybrid TLS-LES method for simulation of high Re complex flows.

The hybrid TLS-LES method additively blends TLS [7] and LES [8] models in a manner similar to the hybrid RANS-LES [3] formulation. In TLS, a flow variable is decomposed first into its large-scale (LS) and small-scale (SS) components. Afterward, a system of governing equations for these components is derived. The model explicitly solves for the SS equations within the LS grid along three one-dimensional (1D) orthogonal lines. In contrast to LES, where the major effort is concentrated on modeling the subgrid-scale terms, focus of TLS is on the modeling of the SS velocity itself. Since, TLS does not employ the spatial filtering approach, therefore, many of the limitations faced by LES are avoided. The hybrid TLS-LES method is designed for efficient simulation of high Re practical flows, as a pure TLS model becomes computationally expensive for such flows [4]. It has been used to study wide variety of flows at low to moderately high Re, such as wall-bounded flows, separating/reattaching flows and wake flows [4, 12, 13]. All of these studies have used the baseline hybrid TLS-LES formulation without adjusting the underlying closure. However, the SS modeling assumptions have been validated in the past only for isotropic turbulence and attached flows [7]. Some of these assumptions, such as ignoring the SS pressure gradient term and the non-linear advection term in the transverse direction to the 1D line for small scales, may affect the dynamics of separating/reattaching and spatially evolving turbulent flows at high Re, and therefore, validity of these assumptions need to be established further so that if needed, TLS model can be improved.

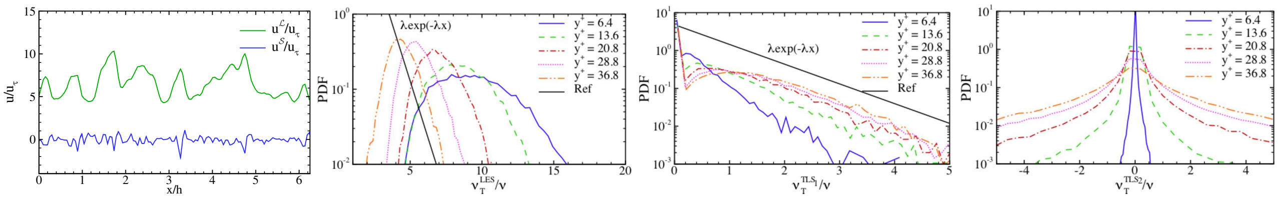
The objective of the present study is to first analyze the modeling assumptions employed by the TLS model by performing a priori analysis of the available DNS data for flow in a periodic channel and flow past a bump placed on the lower surface of a channel. The unique ability of the TLS model to explicitly provide the SS and LS fields can be used to analyze predictions of the TLS model for specific turbulence physics such as presence of co- and counter-gradient diffusion and role of forward or backscatter of energy. In addition, further investigation can be performed to verify some of the assumptions employed by the conventional turbulence closures such as whether use of a single eddy viscosity is sufficient enough for high Re complex flows. Such a posteriori analysis is performed by simulating high Re flows at realistic Reynolds numbers with increasing degree of geometrical complexity. These cases include flow in a periodic channel, flow past a bump placed on the lower surface of a channel and flow past a finite-span airfoil.

PRELIMINARY RESULTS AND FUTURE WORK

The details of mathematical formulation and numerical implementation of the hybrid TLS-LES method is provided elsewhere [12]. The method is well established to perform DNS, LES, TLS or hybrid TLS-LES studies [4, 12]. Here, we describe some preliminary posteriori analysis results for fully developed turbulent flow in a periodic channel.

The flow in a periodic channel is simulated at a friction Reynolds number $Re_\tau = u_\tau h/\nu = 2000$, where h is the channel half-height, u_τ is the friction velocity and ν is the kinematic viscosity. The computational grid is uniform in the streamwise and spanwise directions, and a nominally stretched grid is used in the wall-normal direction. The LS and SS grid resolution is chosen based on the previous studies by the hybrid TLS-LES method [12]. In particular, the LS streamwise and spanwise resolution in terms of wall units is approximately 50 and 25, respectively, whereas the SS resolution is 5 and 2.5 wall units. The minimum wall-normal LS and SS resolution is 6 and 0.5 wall units, respectively. Clearly, the SS resolution is comparable to that used by a DNS. The blending of TLS and LES models is performed by a wall-normal coordinate based static blending function, where the model smoothly switches from TLS to LES within 200 wall units away from the wall.

Figure 1(a) shows the streamwise variation of the LS and SS streamwise velocity fields at $y^+ = 20$. We can observe



(a) $u^{\mathcal{L}}/u_\tau$, $u^{\mathcal{S}}/u_\tau$ (b) $\nu_{t,1}^{\rm LES}$ (c) $\nu_{t,1}^{\rm TLS}$ (d) $\nu_{t,2}^{\rm TLS}$

Figure 1. Instantaneous streamwise variation of the normalized LS ($u^{\mathcal{L}}/u_\tau$) and SS ($u^{\mathcal{S}}/u_\tau$) streamwise velocity fields at $y^+ = 20$ (b) and PDF of eddy viscosity predicted by LES (b) and TLS (c and d) models.

that the LS field shows fluctuations of LS in nature about the mean value. However, the SS field shows presence of intermittent and SS fluctuations. These variations demonstrate the unique ability of the multi-scale TLS-LES formulation to provide explicit solution of LS and SS fields. Such information can be used to assess ability of the TLS-LES model to predict turbulence physics and additionally, can be used to assess and improve conventional closures. Typically, the well known turbulence closures employ eddy viscosity (ν_t) concept, which in general is a tensorial quantity [9], but typically considered as a single quantity in such closures. In the context of TLS and LES of wall-bounded flows, we can define ν_t in two ways through: $\nu_{t,1}^{\rm LES/TLS} = C_\tau \overline{\Delta}\sqrt{\tau_{kk}^{\rm LES/TLS}/2}$ and $\nu_{t,2}^{\rm TLS} = -\tau_{12}^{\rm LES/TLS}/2\overline{S}_{12}$, where ν_t, $\tau_{ij}^{(.)}$ and $\overline{S}_{ij}$ denote eddy viscosity, subgrid stress and resolved strain-rate, respectively, C_τ is a dynamically determined model parameter and $\overline{\Delta}$ is the LES filter size. The first definition of ν_t is typically used by the conventional LES formulation, whereas the second definition is appropriate for planar wall-bounded flows. Figure 1(b-d) show the probability density function (PDF) of ν_t obtained from LES and TLS models at different wall-normal locations. Based on the first definition, ν_t predicted by LES and TLS models shows an exponential distribution and is always positive, thus demonstrating consistency of these models. However, based on the second definition, ν_t predicted by TLS shows presence of positive and negative values, indicating presence of both co- and counter-gradient turbulent diffusion, respectively. The analysis will be extended further to investigate inter-scale energy transfer for flows at realistic Re with increasing geometrical complexity to understand role of forward and backscatter. Note that if the small-scales contain energy containing eddies then there can be significant backscatter of energy from small to large scales [10].

Further investigation of the hybrid TLS-LES method to verify the modeling assumptions, will be conducted by analyzing the DNS data corresponding to flow in a periodic channel and flow past a bump at $Re_\tau = 395$. While the first flow is a canonical example of turbulent wall-bounded flow, the second case is an example of a weakly separating/reattaching flow with presence of curvature effects and a varying streamwise pressure gradient [1]. The posteriori analysis will be extended further to complex flows, which include flow past a bump at $Re_\tau \approx 6500$ [1] and flow past a NACA0015 airfoil at $Re = 1.95 \times 10^6$ [11]. Note that high Re experimental data for validation is available for these cases.

References

[1] A. Bernard, J. M. Foucaut, P. Dupont, and M. Stanislas. Decelerating boundary layer: A new scaling and mixing length model. *AIAA J.*, **41**(2):248–255, 2003.

[2] C. Fureby, N. Alin, N. Wikstrom, S. Menon, N. Svanstedt, and L. Persson. On large eddy simulation of high Reynolds number wall bounded flows. *AIAA J.*, **42**:457–469, 2004.

[3] M. Germano. Properties of the hybrid RANS/LES filter. *Theoret. Comput. Fluid Dynamics*, **17**:225–231, 2004.

[4] A. G. Gungor and S. Menon. A new two-scale model for large eddy simulation of wall-bounded flows. *Progress in Aerospace Sciences*, **46**:28–45, 2010.

[5] F. Hamba. An attempt to combine large eddy simulation with the $k - \epsilon$ model in a channel-flow calculation. *Theoret. Comput. Fluid Dynamics*, **14**:323–336, 2001.

[6] T. Hughes, L. Mazzei, and A. Oberai. The multiscale formulation of large-eddy simulation: Decay of homogeneous isotropic turbulence. *Physics of Fluids*, **13**(2):505–512, 2001.

[7] K. Kemenov and S. Menon. Explicit small-scale velocity simulation for high-Re turbulent flows. *J. Comp. Phys.*, **220**:290–311, 2006.

[8] W.-W. Kim and S. Menon. An unsteady incompressible Navier-Stokes solver for large-eddy simulation of turbulent flows. *International Journal of Numerical Fluid Mechanics*, **31**:983–1017, 1999.

[9] C. Meneveau and J. Katz. Scale-invariance and turbulence models for large-eddy simulation. *Annual Review of Fluid Mechanics*, **32**:1–32, 2000.

[10] S. Menon, P.-K. Yeung, and W.-W. Kim. Effect of subgrid models on the computed interscale energy transfer in isotropic turbulence. *Computers and Fluids*, **25**(2):165–180, 1996.

[11] R. A. Piziali. 2-D and 3-D oscillating wing aerodynamics for a range of angles of attack including stall. *NASA Technical Memorandum*, 1994.

[12] R. Ranjan and S. Menon. A multi-scale simulation method for high Reynolds number wall-bounded turbulent flows. *J. Turbulence*, **14**:1–38, 2013.

[13] R Ranjan and S Menon. On the application of the two-level large-eddy simulation method to turbulent free-shear and wake flows. *J. Turbulence*, **16**(2):136–166, 2015.