PREFERRENTIAL CONCENTRATION OF PARTICLES IN PROTOPLANETARY NEBULA TURBULENCE

Thomas Hartlep[1,2] & Jeffrey N. Cuzzi [2]

[1]*Bay Area Environmental Research Institute, Petaluma, CA, USA*
[2]*NASA Ames Research Center, Moffett Field, CA, USA*

Abstract

Preferential concentration in turbulence is a process that causes inertial particles to cluster in regions of high strain (in-between high vorticity regions), with specifics depending on their stopping time or Stokes number. This process is thought to be of importance in various problems including cloud droplet formation and aerosol transport in the atmosphere, sprays, and also in the formation of asteroids and comets in protoplanetary nebulae.

In protoplanetary nebulae, the initial accretion of primitive bodies from freely-floating particles remains a problematic subject. Traditional growth-by-sticking models encounter a formidable "meter-size barrier" [1] in turbulent nebulae. One scenario that can lead directly from independent nebula particulates to large objects, avoiding the problematic m-km size range, involves formation of dense clumps of aerodynamically selected, typically mm-size particles in protoplanetary turbulence. There is evidence that at least the ordinary chondrite parent bodies were initially composed entirely of a homogeneous mix of such particles generally known as "chondrules" [2]. Thus, while it is arcane, turbulent preferential concentration acting directly on chondrule size particles are worthy of deeper study. Here, we present the statistical determination of particle multiplier distributions from numerical simulations of particle-laden isotropic turbulence, and a cascade model for modeling turbulent concentration at lengthscales and Reynolds numbers not accessible by numerical simulations. We find that the multiplier distributions are scale dependent at the very largest scales but have scale-invariant properties under a particular variable normalization at smaller scales.

CASCADE MODEL AND MULTIPLIER DISTRIBUTIONS

The spatial distribution of particle concentration can be captured statistically by a cascade model [3,4] which predicts the probability distribution functions (pdfs) for dense particle clumps; these PDFs are essentially volume fractions in the nebula having the necessary properties (solids mass and local vorticity) for planetesimal formation. A cascade model presumes that, as energy flows from large eddies to smaller ones, particles and fluid properties are partitioned unequally at each "level" of the cascade from "parent" into "daughter" eddies. The probability distribution functions of the so-called "multipliers" by which particle and fluid properties are partitioned have widths that are parameterized by the parameter β; smaller β means larger width, or a higher probability of strongly asymmetrical partitioning which, repeated over a number of levels, leads to more intermittent local values of particle and fluid properties.

The wide range of scales between integral scale L and Kolmogorov scale η is loosely called the inertial range; in this range the equations of motion are scale-free, and the multiplier pdfs for dissipation of turbulent kinetic energy are known to be scale-independent in atmospheric flows from hundreds to thousands of Kolmogorov lengthscales [5]. This fact motivated [3,4] to develop a cascade model in which the multiplier pdfs for particle concentration were also independent of turbulent eddy lengthscale. Subsequent disagreement with results of others led us to a more in-depth study using highly resolved fluid simulations [6,7] that revealed a scale dependence of multiplier pdfs, which will affect the formation rate and masses of primary planetesimals in the scenario of [3,8].

DETERMINATION OF MULTIPLIER DISTRIBUTIONS

We obtained a dataset of particle trajectories in direct simulations covering 6 large eddy turnover times (T_L) in high Re isotropic turbulence, for $2-6 \times 10^5$ "inertial" particles at each of a range of stopping times ($t_s > 0$), and for fluid tracers ($t_s = 0$). The original simulation was on a cube of 2048η on a side with an integral scale of $L \approx 1024\eta$, and a Taylor-scale Reynolds number $Re_\lambda \approx 400$. The data showed a clear inertial range [6] extending over tens to nearly 1000η based on the second order velocity structure function. The particle locations were spatially binned into snapshots in time. We then determined the multiplier pdfs in the manner described by [4], but now over a range of spatial scales between roughly $12-512\eta$ and a range in Stokes number $St = t_s/t_\eta = 0.6-70$, roughly corresponding to 1mm - 10cm particle diameter under nebula conditions in the asteroid belt region. The dissipation ϵ was known at the locations of the tracers, so its multiplier pdfs could also be determined as a function of spatial binning scale. The spatial distribution of the St = 0 tracers agreed well with a Poisson distribution, as they do not undergo preferential concentration.

The number of inertial and tracer particles in this dataset was relatively small. For example, the average number of particles in a 32η-scale grid cell was only about 2.3. This led to several small-number-statistics sampling biases that needed to be allowed for. These biases led to artificial intermittency; that is, the widths of the apparent multiplier pdfs were larger than their true values. This effect becomes worse at smaller scales where we typically find fewer particles per bin. To correct for these effects, we ran a number of Monte Carlo experiments; assuming that the pdfs are β distributions, we randomly drew samples from the distribution and determined the apparent width σ of the multiplier distribution

depending on N (the number of particles per bin), n (the number of bins) and the value of β of the underlying distribution. Examples of these experiments are shown in figure 1. For instance, very broad pdfs (small β) are not biased as much by this effect as very narrow ones. We have built a large database of such models for a representative range of n, N and values of β from 1 to 5,000, and trained a machine learning algorithm (vector support regression) to predict the true value of β and an error estimate from n, N and the measured σ.

Using this methodology, we were able to derive unbiased estimates of the multiplier distribution β value from the numerical simulation down to bin sizes of 12η. Figure 2 shows our results for the width of the multiplier pdfs, binned on a wide range of lengthscales r given in units of η. In this figure, the particle stopping time t_s has been normalized to a scale-dependent value St_r using the inertial range timescale ratio $(r/\eta)^{-2/3}$, and the constant parameter $St = t_s/t_\eta$ where t_η is the Kolmogorov eddy time. In this plot, a value of $St_r = 1$ represents a particle having stopping time equal to the eddy time for the spatial scale r. Except at the largest binning scales ($r \geq 128\eta$), this scaling collapses the results to a universal family of curves that allows predicting the value of β for any combination of lengthscale and particle stopping time. The minimum near $St_r = 0.2$ suggests that particles with stopping time somewhat shorter than the eddy time on lengthscale r are most strongly concentrated within spatial averages over lengthscale r. This scaling collapses the multipliers much better than an alternate scaling found by [9] which collapses the normalized concentration pdfs themselves; the concentration pdfs can be thought of as the cumulative result of multipliers acting sequentially over all larger scales. After a number of eddy bifurcations to smaller scale, particles of a small but finite range of sizes might thus end up concentrated within a single clump, which may be consistent with new determinations of chondrule size distributions which find them to be broader than previously believed. Our model will ultimately make testable predictions along these lines. However, the curves for the largest lengthscales (r > L/10) diverge from this scale-invariant curve, probably because, in the top decade of lengthscales, turbulent stretching and vortex tube formation have not yet reached their fully developed state. For these largest scales, multiplier statistics are better characterized by their scale as a fraction of L rather than as a multiple of η.
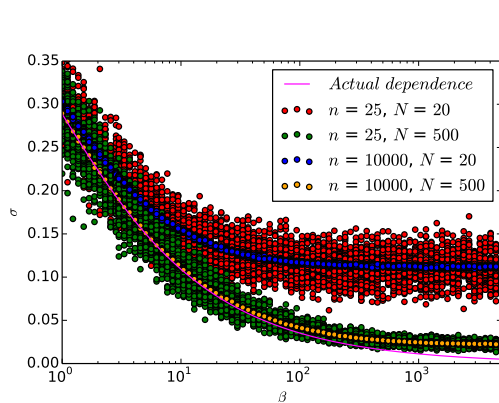

Figure 1. Statistical modeling of the multiplier distribution width σ depending on the number of bins, n, and the number of particles per bin, N. The magenta line denotes the actual width (standard deviation) of the β distributions.
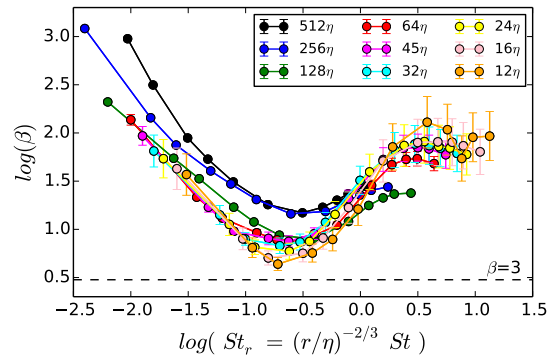

Figure 2. β of multiplier pdfs for particle concentration, plotted as a function of St_r (stopping time t_s normalized by the turnover time of an eddy with scale r), for a variety of binning scales r ranging from 12η to $512\eta \approx L/2$. With this scaling, the multiplier pdf β values collapse to a universal curve except at the largest scales $r > L/10$. A universal curve allows our cascade models to be extended to scales smaller than resolvable by current numerical models.

In summary, these results suggest a universal cascade which has level-dependent properties - but not complicated ones - at the largest decade of scales, but scale-invariant properties under a particular variable normalization at smaller scales. We are now in the process of testing this cascade approach. A version of it may allow predictions to be made in a number of applications where the Reynolds number far exceeds values attainable in numerical or laboratory experiments. Uncertainties remain however, such as the degree to which our asymptotic values of β themselves are Re-dependent. These uncertainties may be resolved by ambitious, but not impossible, simulations at Re-values larger by a factor of 3-10 than those done so far.

References

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