SHAPE OPTIMIZATION OF THE MAXIMIZING PROBLEM OF THE DISSIPATION ENERGY AND ITS EFFECT ON HYDRODYNAMIC STABILITY

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Abstract This paper presents a numerical result for generalized eigenvalue problems on the optimum shape using the numerical method for a flow-field shape optimization problem. The main shape optimization problem addressed in this paper is defined as a two-dimensional lid-driven cavity flow. As an objective cost function, we use the dissipation energy. The domain volume is used as a constraint cost function. The shape derivative of the objective cost function with respect to the domain variation is evaluated using the solution of the main problem and the adjoint problem. Numerical schemes used to conduct the shape optimization problem using an iterative algorithm based on the traction method for reshaping are presented, where the shape of the boundaries aside the top boundary is optimized. Furthermore, by operating a generalized eigenvalue problem, linear neutral curves between the initial domain Ω_0 and the optimum shape domain Ω_1 are compared. Numerical results reveal that the shape is optimized by satisfying the volume constraint. Based on generalized eigenvalue problems, the critical Reynolds number of the optimum shape Ω_1 is larger than that of the initial shape Ω_0.

INTRODUCTION AND FORMULATION OF THE PROBLEM

A shape optimization problem is generalized as a problem of optimizing the boundary design of a domain in which a boundary value problem of partial differential equations is defined, where the domain topology is fixed as that of the initial domain. In this problem, cost functions are defined as functionals of the domain and the solution of the boundary value problem. Especially for fluid dynamics, a method of evaluating the shape derivative for an isolated body in the viscous fluid was presented by Pironneau [1]. And in recent years, E. Katamine et al. [2, 3] demonstrated a shape optimization problem of an isolated body in a domain defined as stationary Navier-Stokes equations, where the dissipation energy is used as the cost function, and is expressed as $\int_\Omega \nabla \boldsymbol{u} \cdot \nabla \boldsymbol{u} d\Omega$. With increasing Re, a linear disturbance becomes unstable in case of Re>Re_c. As a result, the stability of the solution of the stationary Navier–Stokes equations can break down. By the way, the maximizing problem of the dissipation energy is expected to make a linear disturbance more stable. Herein, the maximizing problem of the dissipation energy under the volume constraint is operated to increase Re_c. The two-dimensional lid-driven cavity flow is used as the domain of interest for the first trial, where u=$16x^2(x\text{-}1)^2$, v=0 is defined on the top boundary and u=0, v=0 for the boundary except the top boundary which is the design boundary.

NUMERICAL SCHEMES AND RESULTS

As described herein, FreeFEM++ is used for the following calculations. Furthermore, regarding the numerical procedure, first, the stationary Navier–Stokes equation and continuity equation are solved using the Newton method. Second, the corresponding adjoint equations are solved using the conjugate gradient method. Third, the traction method is used to obtain the domain variation $\boldsymbol{\psi}$ under the volume constraint. Finally, using calculated $\boldsymbol{\psi}$, the domain is reshaped [4, 5] by $\boldsymbol{x}+\boldsymbol{\psi}$, where x denotes the position vectors. For this study, we use a finite-element model with the number of nodes and elements (8281, 16200).

Based on numerical calculations of the shape optimization problem, Fig. 1 shows the optimum domain Ω_1, where Ω_1 is obtained by being optimized at Re=1, and next at Re=11500 using the domain at Re=1 as the initial domain. As a result of the shape optimization problem, the side walls are moved in the inward directions. Furthermore, generalized eigenvalue problems from Re=9500 to 12500 are operated to depict linear neutral curves for the initial shape Ω_0 and the optimum shape Ω_1, as presented in Fig. 2. As figure shows, the critical Reynolds number of the optimum shape Ω_1 is larger than that of the initial shape Ω_0. Fig. 3 and Fig. 4 show stream functions of the perturbations and eigenvalues of the initial shape Ω_0 and the optimum shape Ω_1 at Re=11500.

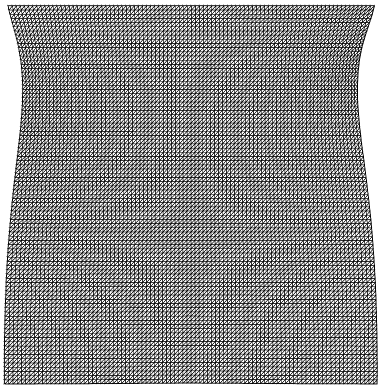


Figure 1.
Domain shape: optimum shape Ω_1.

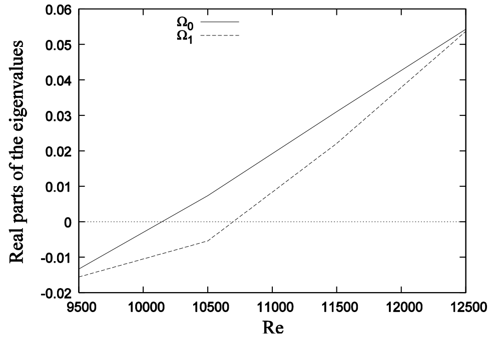
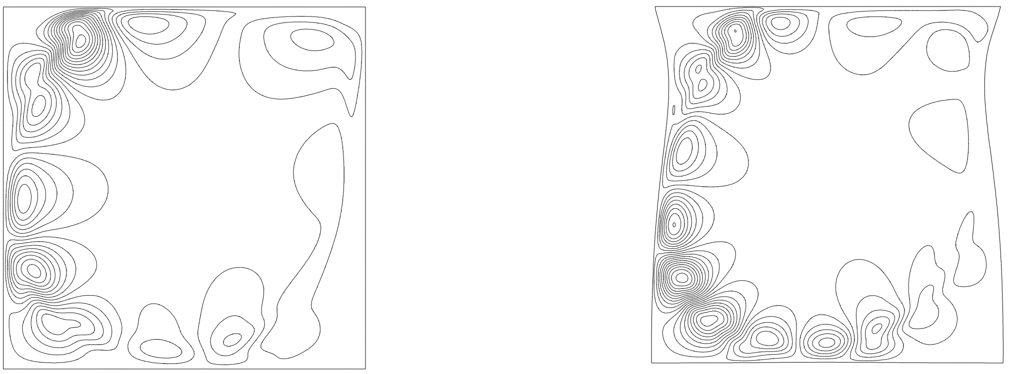


Figure 2.
Comparison of linear neutral curves on the initial shape Ω_0 and the optimum shape Ω_1.



(a) the initial shape Ω_0 (b) the optimum shape Ω_1

Figure 3. Stream functions of the initial shape Ω_0 and the optimum shape Ω_1 at Re=11500.

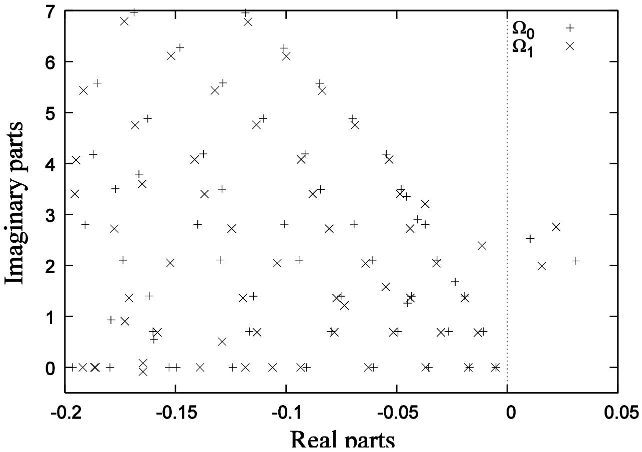


Figure 4. Eigenvalues of the initial shape Ω_0 and the optimum shape Ω_1 at Re=11500.

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