THE TURBULENT DISSIPATION RATE FROM PIV MEASUREMENTS

Guus Bertens, Denis van der Voort, & Willem van de Water
Department of Applied Physics, Eindhoven University of Technology, Eindhoven, the netherlands

Abstract The result of a particle-image velocimetry (PIV) measurement is a velocity field averaged over interrogation windows. This severely affects the measurement of small-scale turbulence quantities when the interrogation window size is much larger than the smallest length scale in turbulence, the Kolmogorov length. In particular, a direct measurement of the dissipation rate demands the measurement of gradients of the velocity field, which are now underestimated because the small-scale motion is not resolved. A popular procedure is to relate the statistical properties of the measured, but underresolved gradients to those of the true ones, invoking a large-eddy argument [3]. We show that the used proportionality constant, the Smagorinsky constant, should depend on the window overlap, on the used elements of the strain tensor, and on the way in which derivatives are approximated.

LARGE-EDDY PIV

The result of particle-image velocimetry is a velocity field which is averaged over the interrogation window. This strongly affects the measurement of small-scale quantities in turbulent flows. For example, the dissipation rate $\epsilon = \nu \sum_{i,j} \left\langle (\partial u_i / \partial x_j)^2 \right\rangle$, where ν is the kinematic viscosity, involves the sum of squared derivatives of the components u_i of the velocity field. It requires the resolution of the velocity field down to the Kolmogorov scale η, where the velocity field is smooth. Normally, the Kolmogorov length η is much smaller than the linear dimension L of the interrogation window, so that the magnitude of derivatives is under-estimated. Also, addition, an estimate of those derivatives involves finite differences of the averaged velocity field in neighboring interrogation windows, so that the estimate depends on the degree of overlap of these windows, and on the way in which derivatives are approximated by finite differences.

In *large-eddy PIV* it is assumed that the statistics of the averaged velocity field is universal, with a universal relation between the measured derivatives of the averaged field and the true dissipation rate, ϵ_{LE} [3],

$$\epsilon_{\mathrm{LE}} = 2^{3/2}\, C_{\mathrm{Sm}}^2\, L^2\, \left\langle S^3 \right\rangle , \tag{1}$$

with $S = (S_{ij} S_{ij})^{1/2}$, $S_{ij} = \frac{1}{2}(\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_j)$, L the size of the interrogation window and $\bar{u}$ the velocity field averaged over an interrogation window. The commonly used value of the Smagorinsky constant is $C_{\mathrm{Sm}} = 0.17$ [3, 1], but it should depend on the degree of window overlap, on the way in which derivatives are approximated and on which components of the strain tensor are used in the estimate of S. In fact, the commonly used value of the Smagorinsky constant $C_{\mathrm{Sm}} = 0.17$ is based on a box filter in *Fourier space*, contrary to the *real space* box filter that is associated with PIV.

In planar PIV, not all velocity gradients are acessible, but the missing ones can be guessed on the basis of isotropy and incompressibility. Therefore, only $\langle S^2 \rangle$ can be estimated, and not $\langle S^3 \rangle$, and the assumption $\langle S^3 \rangle = \langle S^2 \rangle^{3/2}$ is unavoidable.

Using a three-dimensional model spectrum (the Pao spectrum, [2]), which is characterized by the dissipation rate ϵ_0 and the Reynolds number, it is possible to analytically derive the influence of averaging on the measured S, and thus to check the experimental procedure embodied by Eq. 1. For the measured one-dimensional spectrum and the second-order structure function the result shown in Fig. 1.

DERIVATIVES FROM FINITE DIFFERENCES

Planar PIV provides averaged velocity fields on a discrete grid of overlapping interrogation windows, with $0 < \alpha < 1$ the overlap factor, such that for $\alpha = 0.5$ windows overlap 50%, and 75% for $\alpha = 0.25$. Derivatives may be approximated by central differences (CD):

$$\partial u / \partial x \approx \left[\bar{u}(x + \alpha L, y) - \bar{u}(x - \alpha L, y) \right] / 2\alpha L, \tag{2}$$

or with a least-squares approach (LS),

$$\partial u / \partial x \approx \left[2\bar{u}(x + 2\alpha L, y) + \bar{u}(x + \alpha L, y) - \bar{u}(x - \alpha L, y) - 2\bar{u}(x - 2\alpha L, y) \right] / 10\alpha L. \tag{3}$$

These choices severely affect the outcome of the large-eddy PIV procedure and the value of the Smagorinky constant. In addition, they also affect the apparent small-scale anisotropy of the turbulent flow. For isotropic and incompressible

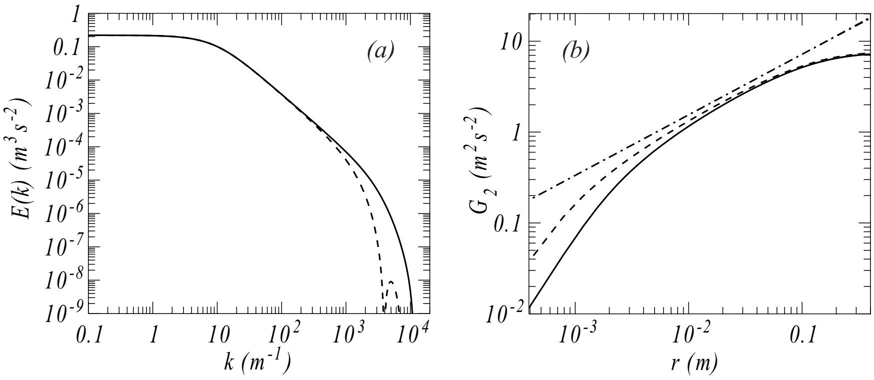

Figure 1. (a) Influence of interrogation window averaging on the energy spectrum. Input parameters are $u = 1.9\ \mathrm{m/s}$, $\eta = 8.7 \times 10^{-5}\ \mathrm{m}$, $\epsilon_0 = 60\ \mathrm{m^2 s^{-3}}$, and window size $L = 1.6 \times 10^{-3}\ \mathrm{m}$. These numerical values are from the experiment which will be discussed in the presentation. (b) Influence of interrogation window averaging on the second-order structure function. The full line shows the structure function of the averaged velocity field, the dashed line that of the bare, unaveraged velocity field, and the dash-dotted line the Kolmogorov prediction $G_2(r) = C_2 \epsilon^{2/3} r^{2/3}$, with $C_2 = 2.12$. For the averaged velocity field, an inertial range is hardly recognizable.

turbulence $\langle (\partial u / \partial y)^2 \rangle = 2 \langle (\partial u / \partial x)^2$, but this no longer holds for the discretized derivative of averaged velocity fields. The measured anisotropy now depends on the size of the interrogation window L, on the overlap factor α and on the approximation to the derivative.

The large-eddy correction to the measured dissipation rate, $\epsilon_{\mathrm{LE}}/\epsilon_0$ is shown in Fig. 2. It is almost independent of the interrogation window size, which, of course is the essence of the large-eddy idea. Surprisingly, almost no correction is needed for half-overlapping windows, and the central difference approximation of the derivative.

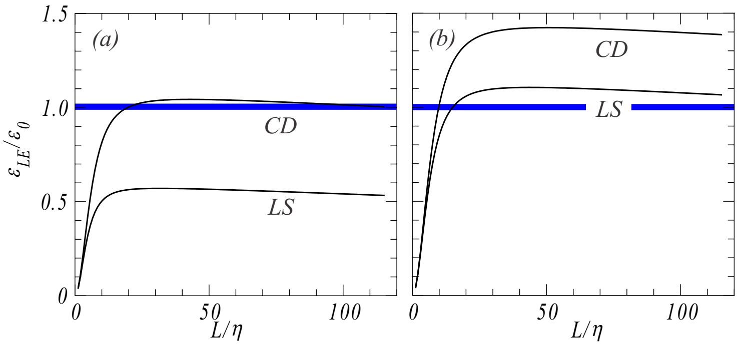

Figure 2. Large-eddy corrected ϵ_{LE} as a function of interrogation window size L for two different values of the window overlap, $\alpha = 0.5$ (a) and $\alpha = 0.25$ (b). We have used the standard value of the Smagorinsky constant $C_{\mathrm{Sm}} = 0.17$. The thick blue lines indicate $\epsilon_{\mathrm{LE}} = \epsilon_0$, with ϵ_0 the input dissipation rate. The lines indicated by "CD" are computed using the central difference approximation to the derivative (Eq. 2), those marked by "LS" use Eq. 3. Almost no correction is needed for $\alpha = 0.5$ in combination with the central difference approximation.

In conclusion, large-eddy PIV works, but the Smagorinsky constant should depend on the used PIV procedure, and on the way in wich derivatives are approximated.

References

[1] P. Lavoie, G. Avallone, F. De Gregorio, G.P. Romano, and R.A. Antonia. Spatial resolution of PIV for the measurement of turbulence. *Experiments in Fluids*, **43**(1):39–51, 2007.

[2] S.B. Pope. *Turbulent Flows*. Cambridge University Press, 2000.

[3] J. Sheng, H. Meng, and R.O. Fox. A large eddy PIV method for turbulence dissipation rate estimation. *Chemical Engineering Science*, **55**(20):4423 – 4434, 2000.