

**DNS OF TURBULENT MIXING LAYERS WITH VARIABLE DENSITY**

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**Abstract** We present some preliminary results of direct numerical simulations of three-dimensional, temporal, plane mixing layers with variable density. The simulations are run with a parallel in-house code that solves the Navier-Stokes equations in the Low-Mach number approximation, using a novel algorithm based on an extended version of the velocity-vorticity formulation used by Kim, Moin & Moser (1987) for incompressible flows. The simulations are run with  $Pr = 0.7$  and achieve  $Re_\lambda = 90 - 110$  during the self-similar evolution of the mixing layer. Four cases with density ratios  $s = 1, 2, 4$  and  $8$  are presented. Our results show good agreement with previous experimental and numerical studies, and allow us to characterise the scales in the temperature spectra.

**INTRODUCTION**

Variable density turbulence is an important flow regime in many engineering applications. A typical example is combustion in diffusion flames [1], but it is also important in a variety of geophysical flows. In all these cases, the velocity in the fluid is much smaller than the speed of sound (Low Mach number range [2]). As a consequence, the thermodynamic pressure is essentially constant, and density is inversely proportional to temperature. Because of this, variable density turbulence retains some of the properties of incompressible flows, although the density variations modify the momentum and energy cascades in ways that are not yet fully understood. In order to analyse this problem, we present here Direct Numerical Simulations of temporal mixing layers between two streams of different densities. The simulations are carried out using a novel algorithm, that combines a Helmholtz decomposition of the momentum vector with the velocity-vorticity formulation of the momentum equations [3]. The cases presented here are run in relatively large computational boxes, achieving  $Re_\lambda = 90 - 110$ . Four density ratios are considered in this study,  $s = 1, 2, 4$  and  $8$ . At the time of writing this abstract, the simulations with density ratios  $s = 1, 2$  and  $4$  are completed, while the simulation with  $s = 8$  is underway.

**FLOW CONFIGURATION AND NUMERICAL METHOD**

The set-up of the simulations consists of a three-dimensional temporally evolving mixing layer between two streams of different densities separated in the vertical direction ( $y$ ). The temporal mixing layer is planar, with two homogeneous directions ( $x$  streamwise,  $z$  spanwise). Initially, there is a jump between the velocity and density of the upper ( $y > 0$ ) and lower ( $y < 0$ ) streams, which defines the characteristic velocity of the mixing layer and its density ratio,  $s$ . Note that gravity effects are not included in these simulations.

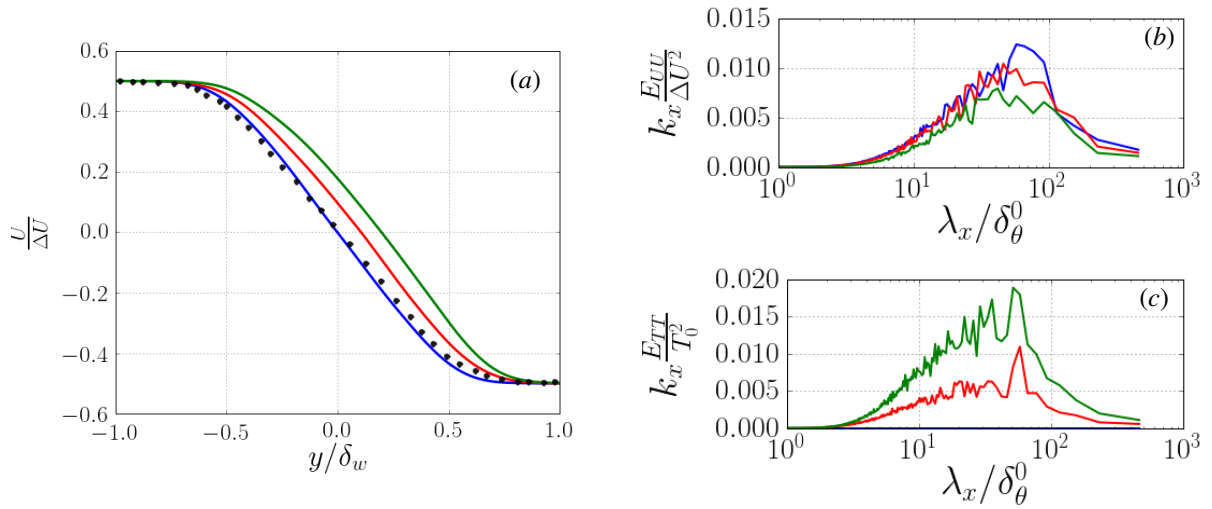
This mixing layer is simulated using an in-house code that solves the Low-Mach number approximation of the Navier-Stokes equations. In order to solve these equations in a numerically efficient way, we apply a Helmholtz decomposition to the momentum vector,

$$\rho \vec{u} = \vec{m} + \nabla \psi, \quad (1)$$

where  $\vec{m}$  is a solenoidal (divergence-free) component and  $\nabla \psi$  is an irrotational (curl-free) component. Following [3], the momentum equation is expressed in terms of two evolution equations: one for the laplacian of the vertical component of the solenoidal term,  $\phi = \nabla^2 m_y$ , and a second equation for the vertical component of the curl of the momentum vector,  $\Omega = [\nabla \times \rho \vec{u}]_y$ . These equations are supplemented with two additional momentum equations for the time evolution of the averaged value of streamwise and spanwise momentum over the homogeneous directions ( $\langle \rho u \rangle$ ,  $\langle \rho w \rangle$ ), as well as the energy and mass conservation equations.

These six equations are integrated numerically using a scheme similar to [4]: momentum and energy equations are time advanced first, to update  $\phi$ ,  $\Omega$ ,  $\rho$ ,  $\langle \rho u \rangle$  and  $\langle \rho w \rangle$ . Then, the mass conservation equation is used to update  $\psi$ . This algorithm is implemented using a three-stage low-storage Runge-Kutta. Since the horizontal directions are homogeneous, the spatial discretization uses Fourier expansions in the  $x$  and  $z$  directions, with a 7th order Compact Finite Difference scheme in the vertical direction. The non-linear terms are computed with a pseudo-spectral scheme and a 2/3 dealiasing. The computational domain is  $L_x \times L_y \times L_z = 460\delta_\theta^0 \times 258\delta_\theta^0 \times 172\delta_\theta^0$ , where  $\delta_\theta^0$  is the initial momentum thickness. This domain is discretized with  $1536 \times 1101 \times 576$  collocation points. This corresponds to a resolution of  $\lesssim 3\eta$  in the self-similar range of the evolution of the mixing layer. For reference, the momentum thickness during this self-similar range is  $\delta_\theta \approx 4.5\delta_\theta^0$ . The domain is periodic in  $x$  and  $z$ , and homogeneous boundary conditions ( $m_y = 0$  and  $\Omega = 0$ ) with symmetric vertical entrainment are set in the  $y$  direction.

The flow is initialized as in [5], with a hyperbolic profile for the mean streamwise velocity,  $\langle u \rangle$ , and mean density,  $\langle \rho \rangle$ . The other two mean velocity components are set to zero. Random velocity fluctuations are added with 10% of turbulence intensity.



**Figure 1.** *a)* Mean streamwise velocity profiles, plotted as a function of the vertical coordinate normalized with the vorticity thickness. *b)* Velocity and *c)* temperature premultiplied spectra. In all plots, colours are blue for  $s = 1$ , red for  $s = 2$  and green for  $s = 4$ . Black dots in *a)* representing results from [6] for  $s = 1$ . The data is averaged over a short period of time around  $t \approx 200\delta_m^0/\Delta U$ , which corresponds to the beginning of the self-similar evolution of the mixing layer for all cases.

## RESULTS

Figure 1 shows preliminary results from our simulations. In particular, figure 1*a* includes the mean velocity profile for cases  $s = 1, 2$  and  $4$ . The data is plotted for a time corresponding to the beginning of the self-similar evolution of the mixing layers, when the vorticity thickness of the three cases is approximately the same (within a 3%). For comparison, the mean velocity reported by [6] for an incompressible mixing layer is included, showing good agreement with our corresponding case,  $s = 1$ . As the density ratio increases, we observe a shift in the mean velocity profile, similar to the ones reported elsewhere for compressible mixing layers (see for instance [5]).

Figure 1*b* and *c* show respectively the premultiplied spectra of the streamwise velocity and temperature in the plane  $y = 0$ , as a function of the streamwise wavelength normalised with the initial momentum thickness. It can be observed that the scales appearing in the temperature spectra are similar to the ones in the velocity spectra, although it seems that the temperature spectra is slightly shifted towards smaller scales.

Overall, a good agreement is found between our preliminary results and the results reported in the literature for similar flows. Other statistics, including velocity fluctuation profiles and spectra of other variables, will be discussed in the presentation.

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