DISPERSIVE TO NONDISPERSIVE TRANSITION IN THE PLANE WAKE AND CHANNEL FLOWS

Francesca De Santi[1], Federico Fraternale[1], Daniela Tordella[1]
[1]*Dipartimento di Ingegneria Meccanica ed Aerospaziale, Politecnico di Torino, Torino, Italy*

Abstract By varying the wavenumber over a large and finely discretized interval of values, we analyse the phase and group velocity of linear three-dimensional travelling waves both in the plane wake and channel flows to get the transition between dispersive and non-dispersive behaviour. The dispersion relation is computed from the Orr-Sommerfeld and Squire eigenvalue problem by observing the least stable mode, see figure 2, panels (**a**,**b**) and the comparison with [1, 2, 4–11, 15, 16]. The group velocity v_g is also shown. The Reynolds number varies in the 20-100, 1000-8000 ranges for the wake and the channel flow, respectively, while we consider wavenumbers in the range 0.1-10. The wake basic flow consists of the first two orders of the Navier-Stokes matched asymptotic expansion described in [3, 13, 14]. At low wavenumbers we observe a dispersive behaviour where the phase speed and the group velocity substantially differ. The relevant perturbed solution is amenable to the typical solution belonging to the left branch of the eigenvalue spectrum, see the two examples shown in figure 1 (channel flow: $Re = 6000,\ k = 1$; wake $Re = 100,\ k = 0.7$).

By rising the wave number value, we observe a sharp transition from the dispersive to the nondispersive regime. This transition is located at a critical wave number k_d which is a function of the Reynolds number Re, the wave angle ϕ, and the wake downstream observation point x_0. Precisely, k_d increases with Re and decreases with ϕ for the wake flow, while these trends are reversed for the channel flow, see tables 1,2. Beyond the wavenumber threshold, the observed least-stable mode belongs to the right branch of the spectrum.

The asymptotic solutions in the dispersive region are *wall modes* for the channel flow , and *in-wake modes* for the wake flow. This means that, for both the flows, the dispersive behaviour is related to perturbations with high momentum variations (high vorticity) in correspondence to the base flow high-shear region. On the contrary, if $k > k_d$ the solutions are *central modes* for the channel case, and *out-of-wake modes* for the wake flow. In these cases, the disturbance has high variations outside the base flow high-shear region.

To understand the physical mechanism of the dispersive-nondispersive transition we focused on time variation of the wave kinetic energy associated to the convective transport. Figure 2 (**c**,**d**) shows the convective term as a function of the wavenumber for the two least stable modes. We observe that the dispersive-nondisperive transition allows waves $k > k_d$ to keep the lowest possible temporal variation of kinetic energy, i.e. the lowest decay. This remains true also when all the other more stable modes are considered. In practice nondispersive waves maintain their convective energy with k.

References

[1] M. Asai and J. M. Floryan. Experiments on the linear instability of flow in a wavy channel. *Eur. J. Mech. B/Fluids*, **25**:971–986, 2006.
[2] D. Barkley. Linear analysis of the cylinder wake mean flow. *Europhys. Lett.*, **75**(750–756), 2006.
[3] M. Belan and D. Tordella. Convective instability in wake intermediate asymptotics. *J. Fluid Mech.*, **552**(127–136), 2006.
[4] F. Giannetti and P. Luchini. Structural sensitivity of the first instability of the cylinder wake. *J. Fluid Mech.*, **581**:167–197, 2007.
[5] N. Ito. *Trans. Japan Soc. Aero. Space Sci.*, **17**:65, 1974.
[6] M. Nishioka, S. Lida, and Y. Ichikawa. An experimental investigation of the stability of plane poiseuille flow. *J. Fluid Mech.*, **72**:731–751, 1975.
[7] M. Nishioka and H. Sato. Measurements of velocity distributions in the wake of a circular cylinder at low reynolds numbers. *J. Fluid Mech.*, **65**:97–112, 1974.
[8] C. Norberg. An experimental study of the flow around a circular cylinder: Influence of aspect ratio. *J. Fluid Mech.*, **258**:287–316, 1994.
[9] P. Paranthoën, L. W. B. Browne, S. LeMasson, F. LeMasson, and J. C. Lecordie. Characteristics of the near wake of a cylinder at low reynolds numbers. *Eur. J. Mech. B/Fluids*, **18**:659–674, 1999.
[10] B. Pier. On the frequency selection of finite-amplitude vortex shedding in the cylinder wake. *J. Fluid Mech.*, **458**:407–417, 2002.
[11] A. Roshko. On the development of turbulent wakes from vortex streets. *NACA*, **1932**, 1954.
[12] P. J. Schmid and D. S. Henningson. *Stability and Transition in Shear Flows*. Springer, 2001.
[13] D. Tordella and M. Belan. A new matched asymptotic expansion for the intermediate and far flow behind a finite body. *Phys. Fluids*, (1897–1906), 15.
[14] D. Tordella, S. Scarsoglio, and M. Belan. A synthetic perturbative hypothesis for multiscale analysis of convective wake instability. *Phys. Fluids*, **18**:054105, 2006.
[15] C. H. K. Williamson. Oblique and parallel modes of vortex shedding in the wake of a circular cylinder at low reynolds numbers. *J. Fluid Mech.*, **206**:579–627, 1989.
[16] A. Zebib. Stability of viscous flow past a circular cylinder. *J. Engin. Maths*, **21**:155–165, 1987.

Table 1. Values of the dispersive regime threshold wavenumber k_d for the channel flow, for different values of Reynolds number and obliquity angles. The uncertainty on k_d due to the discretization is ± 0.005. The reference length is the channel half-width.

Re	$\phi = 0$	$\phi = \pi/6$	$\phi = \pi/4$	$\phi = \pi/3$
1000	2.071	2.111	2.168	2.256
2000	1.883	1.922	1.979	2.073
3000	1.764	1.803	1.866	1.960
4000	1.686	1.725	1.784	1.878
5000	1.623	1.662	1.721	1.815
6000	1.576	1.615	1.670	1.765
7000	1.536	1.568	1.627	1.720
8000	1.497	1.536	1.589	1.682

Table 2. Values of the dispersive regime threshold wavenumber k_d for the wake flow, for different Reynolds numbers, obliquity angles and for a streamwise station $x_0 = 20$. The reference length is the diameter of the bluff-body generating the wake.

Re	$\boldsymbol{\phi = 0}$	$\boldsymbol{\phi = \pi/6}$	$\boldsymbol{\phi = \pi/4}$	$\boldsymbol{\phi = \pi/3}$
20	0.756	0.732	0.691	0.616
30	0.968	0.943	0.896	0.815
40	1.153	1.118	1.086	0.987
50	1.325	1.294	1.250	1.159
60	1.471	1.441	1.400	1.318
70	1.616	1.587	1.550	1.463
80	1.748	1.719	1.686	1.596
90	1.881	1.851	1.809	1.728
100	1.992	1.983	1.945	1.860

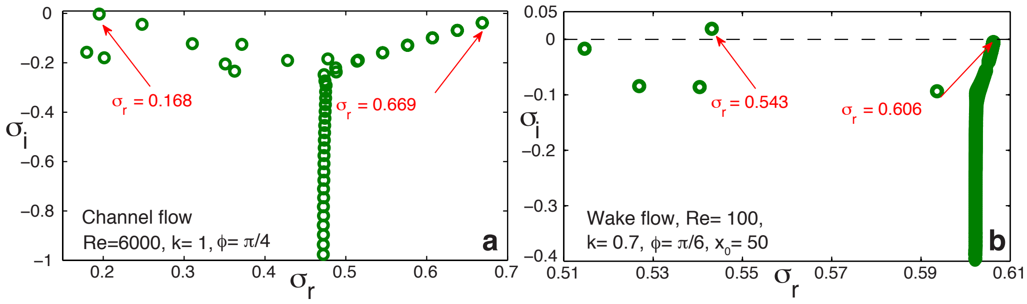

Figure 1. Eigenvalue spectra for a 3D wave inside a subcritical channel flow and a supercritical plane wake flow.

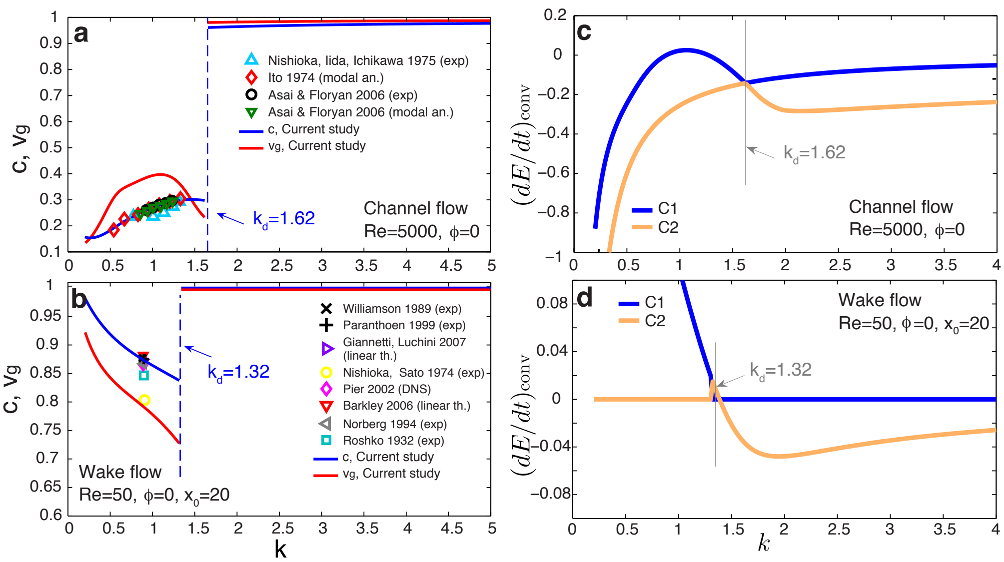

Figure 2. Dispersive/nondispersive behavior. (**a**,**b**) Phase and group velocity as a function of the wavenumber for longitudinal waves ($\phi = 0$) and comparison with literature results: [1, 5, 6] for the channel, [2, 4, 7–11, 15, 16] for the wake. The phase velocity is represented with the blue line, while the group velocity with the red one. This set of literature data includes laboratory experiments, linear modal and global stability analysis, and DNSs. The wavenumber range is discretized with $\Delta k = 0.005$. k_d indicates the threshold value that separates the dispersive waves from the non-dispersive waves. These solutions are computed by both a Galerkin method involving Chandrasekhar normal functions ($N = 250$) and a Cěbyšhëv spectral code [12]. (**c**,**d**) Energy equation convective term $(dE/dt)_C = 1/k^2 \Im \int U'(k\bar{v}\partial_y \hat{v})\, dy$, as a function of the wavenumber, for the 2D case. $U(y)$ is the base flow, $\hat{v}$ the velocity perturbation, $\bar{v}$ is the complex-conjugate. The blue curve represents the trend for the least-stable eigenfunction, while the yellow curve is the trend for the second-last one.