DISPERSION IN UNSTEADY JETS AND PLUMES

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Abstract We investigate the transport of both passive and active scalars in fully developed turbulent axisymmetric jets and plumes using data from direct numerical simulation. In both cases we simulate the response of the flow to an instantaneous increase in the scalar flux at the source and our focus is on the determination of the rate at which the resulting disturbance propagates and spreads in the longitudinal direction. We apply Taylor's theory of shear-flow dispersion [9] to free-shear flows and therefore model the way in which departures from self-similarity result in the longitudinal mixing of integral quantities. The resulting integral models exhibit a good agreement with the simulation data and, in the case of passive scalar transport, admit an analytical similarity solution. For the case of active scalar transport we examine the buoyancy flux in an unsteady plume and show that the momentum–energy framework [7], rather than the classical volume–momentum framework [6], provides the natural setting from which to view the effects of dispersion. Consequently, we demonstrate the effect that dispersion has on turbulent entrainment and the way in which a plume responds to source perturbations in its buoyancy flux.

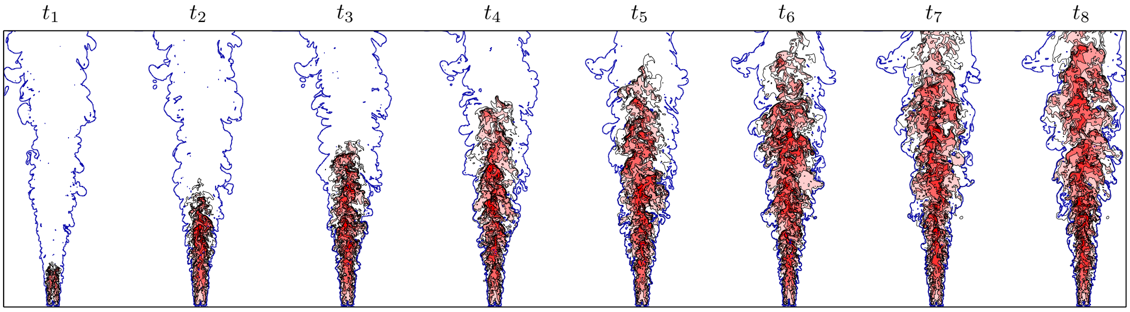


Figure 1: Isoregions of the instantaneous scalar concentration (red) and threshold of the instantaneous enstrophy (blue) at times $t_1, \dots, t_8$. The quantities displayed in the figure are non-dimensionalised using local characteristic scales.

PASSIVE SCALAR DISPERSION IN UNSTEADY JETS

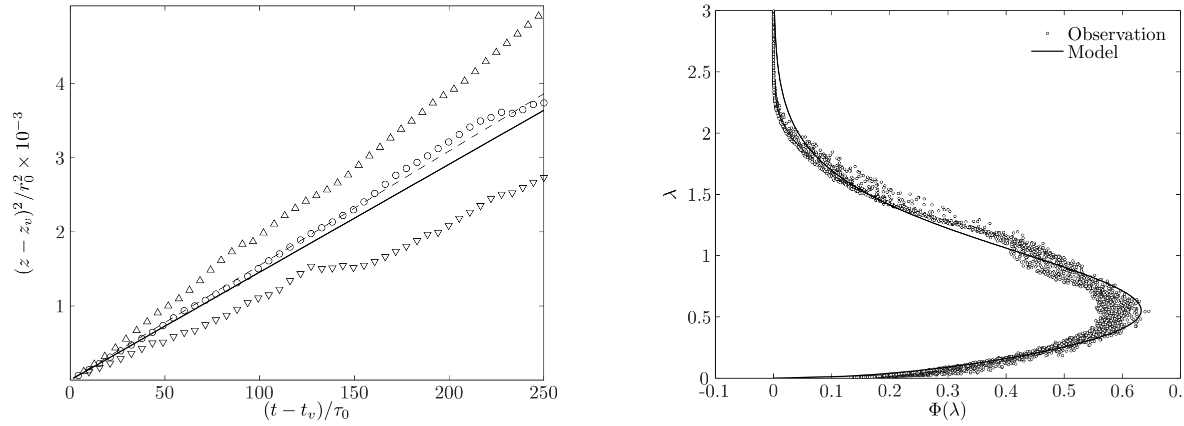
Shear-flow dispersion is a primary source of longitudinal mixing in integral models for passive scalar transport and consequently finds application in many problems involving contaminant transport. However, to date the significant attention directed towards passive scalar transport in turbulent jets has been focused on steady releases; unsteady releases, for which dispersive effects are expected to be significant, have received relatively little attention.

In this work we investigate the transport of a passive scalar in a fully developed turbulent axisymmetric jet at Re = 4815 using data from direct numerical simulation. We simulate the response of the concentration field to an instantaneous increase of the scalar flux at the source (figure 1). To analyse the time evolution of this statistically unsteady process we take an ensemble average over 16 independent statistically equivalent simulations. We find that the evolution of the radial scalar integral C_m is a self-similar process, with front position and longitudinal spread scaling as $\sqrt{t}$ (figure 2, a). The longitudinal mixing of C_m is shown to be primarily caused by shear-flow dispersion. Following recent work into the behaviour of jets whose source momentum flux undergoes a rapid change [1, 2], the classical theory for shear-flow dispersion [9] is applied to passive scalar transport in turbulent jets to obtain a closure that couples the integral scalar flux with the integral concentration C_m [3].

To account for the fundamental difference between a pipe flow, for which the steady-state concentration C_{m0} is everywhere uniform, and a jet, for which the steady-state concentration is non-uniform, we employ a dimensionless concentration $\mathscr{C}_m \equiv C_m/C_{m0}$. In the steady state, the dimensionless concentration $\mathscr{C}_m$ is uniform by construction, and allows us to formulate a dispersion closure for the dimensionless scalar flux θ_m according to

$$\theta_m = \theta_0 - \theta_1 \frac{w_m r_m^2}{\nu_T} \frac{1}{\mathscr{C}_m} \frac{\partial \mathscr{C}_m}{\partial z}, \tag{1}$$

in which $w_m(z)$ is the characteristic longitudinal velocity in the jet, $r_m(z)$ is the characteristic radius of the jet and ν_T is a characteristic eddy viscosity. The constant θ_0 characterises the width of the velocity profile relative to that of the scalar profile and determines the rate at which the scalar propagates in the longitudinal direction. The constant θ_1 accounts for the correlation of the velocity profile with small departures from self-similarity in the scalar profile, and therefore determines the rate at which the scalar mixes in the longitudinal direction. Application of the dispersion closure to a two-dimensional jet results in an integral transport equation that is fully consistent with a recent model of unsteady dispersion



(a)

(b)

Figure 2: (a) Position of scalar front [○], longitudinal extent [△, ▽] and corresponding linear fit (dashed line), compared to the 'top-hat' propagation (solid line). (b) Comparison of observed normalised concentration with the analytical self-similar solution of the dispersion model.

in a planar jet [5]. In the far field, making use of the observed power-law scaling of the jet, the proposed shear-flow dispersion model can be reduced to an ordinary differential equation, whose analytical solution exhibits a good agreement with the simulation data (figure 2, b).

ACTIVE SCALAR DISPERSION IN UNSTEADY PLUMES

Active scalar dispersion is investigated by doubling the source buoyancy flux in an axisymmetric turbulent plume at $Re = 2400$. The step change in the buoyancy flux propagates and spreads in the longitudinal direction, modifying the steady-state volume flux and momentum flux in the plume. We discuss the role of longitudinal turbulence transport and shear-flow dispersion in determining the propagation velocity of the front and its mixing properties. In addition to providing the natural way of understanding shear-flow dispersion, the momentum–energy framework [7], in place of the classical volume–momentum framework [6], exposes features of unsteady plumes that are hidden in the steady state. It reveals, for example, that properties such as entrainment and the radius of the plume depend on the dimensionless energy flux, buoyancy flux, and turbulence production independently.

Our proposed model for unsteady plumes incorporates dispersion of the mean-flow energy integral [2] and buoyancy integral. The model exhibits a good agreement with the DNS results, capturing both the position and longitudinal extent of the front and characteristic features of the plume's leading-order integral quantities. Motivation for this work comes from the recent discovery that several existing unsteady plume models are ill-posed [8] in the absence of an appropriate description of longitudinal mixing. The dispersion closure we present has the desirable feature of being valid for both unsteady plumes and unsteady jets, not modifying the classical solutions to the steady-state plume equations and can be used to model the longitudinal mixing of both the mean kinetic energy and buoyancy.

References

[1] J. Craske and M. van Reeuwijk. Energy dispersion in turbulent jets. Part 1. Direct simulation of steady and unsteady jets. *J. Fluid Mech.*, **763**:500–537, 1 2015.

[2] J. Craske and M. van Reeuwijk. Energy dispersion in turbulent jets. Part 2. A robust model for unsteady jets. *J. Fluid Mech.*, **763**:538–566, 1 2015.

[3] J. Craske, A.L.R. Debugne and M. van Reeuwijk. Shear-flow dispersion in turbulent jets *J. Fluid Mech.*, (submitted).

[4] E. Kaminski, S. Tait, and G. Carazzo. Turbulent entrainment in jets with arbitrary buoyancy. *J. Fluid Mech.*, **526**:361–376, 2005.

[5] J. R. Landel, C. P. Caulfield, and A. W. Woods. Streamwise dispersion and mixing in quasi-two-dimensional steady turbulent jets. *J. Fluid Mech.*, pages 1–47, 2012.

[6] B. R. Morton, G. I. Taylor, and J. S. Turner. Turbulent gravitational convection from maintained and instantaneous sources. *Proc. R. Soc. Lond.* A, **234**(1196):1–23, 1956.

[7] C. H. B. Priestley and F. K. Ball. Continuous convection from an isolated source of heat. *Q. J. R. Meteorol. Soc.*, **81**(348):144–157, 1955.

[8] M. M. Scase and R. E. Hewitt. Unsteady turbulent plume models. *J. Fluid Mech.*, **697**:455–480, 2012.

[9] G. I. Taylor. The dispersion of matter in turbulent flow through a pipe. *Proc. R. Soc. Lond.* A, **223**(1155):446–468, 1954.