PROBING TURBULENCE INTERMITTENCY VIA AUTO-REGRESSIVE MOVING-AVERAGE MODELS

Davide Faranda[1], Flavio Maria Emanuele Pons [2], François Daviaud [1] & Bérengère Dubrulle[1]
[1]*Laboratoire SPHYNX, SPEC, CEA Saclay, CNRS UMR 3680, 91191 Gif-sur-Yvette, France*
[2]*Department of Statistics, University of Bologna, Via delle Belle Arti 41, 40126 Bologna, Italy.*

Abstract We suggest a new approach to probing intermittency corrections to the Kolmogorov law in turbulent flows based on the Auto-Regressive Moving-Average modeling of turbulent time series. We introduce an index Υ that measures the distance from a Kolmogorov-Obukhov model in the Auto-Regressive Moving-Average models space. Applying our analysis to Laser Doppler Velocimetry measurements in a von Kármán swirling flow, we show that Υ is proportional to traditional intermittency corrections computed from structure functions. Therefore it provides the same information, using much shorter time series. We conclude that Υ is a suitable index to reconstruct intermittency in experimental turbulent fields.

INTRODUCTION

One of the few exact results known for isotropic, homogeneous and mirror-symmetric turbulence is the 4/5 - law derived by Kolmogorv in 1941. It links the longitudinal velocity increments $\delta u_\ell = u(x+\ell) - u(x)$ to the mean rate of energy dissipation $\langle\epsilon\rangle$ via $\langle\delta u_\ell^3\rangle = -\frac{4}{5}\langle\epsilon\rangle\ell$, where $\langle\rangle$ denotes averaging. This exact relation was generalized by Kolmogorov[9] as a scaling law $\delta u_\ell \equiv (\epsilon\ell)^{1/3}$, where $\equiv$ means *has the same statistical properties*. Should ϵ be a non stochastic constant, the scaling law would imply self-similar behavior for the structure functions of order p, $S_p(\ell) = \langle\delta u_\ell^p\rangle$, that would scale like:

$$F_p(\ell) \sim \epsilon^{p/3}\ell^{p/3}. \tag{1}$$

For $p = 3$, we recover the 4/5 - law. For $p = 2$, this equation predicts a second order structure function that varies like $\ell^{2/3}$. By Fourier transform, this is equivalent to a one dimensional energy spectrum scaling with wavenumber k as : $E(k) \sim k^{-5/3}$, also known as the Kolmogorov spectrum. More generally, eq. (1) predicts a linear law for the exponent of the structure functions $\zeta(p) = \mathrm{d}\ln F_p(\ell)/\mathrm{d}\ln\ell = p/3$. However, as pointed out by Landau and recognized by Kolmogorov [9], there is no reason to assume that ϵ is a constant over space and/or time, so that it should rather be viewed as a stochastic process, that depends upon the scale ℓ at which it is measured: $\epsilon \equiv \epsilon(\ell)$. In such a case, the correct scaling of the structure function is rather

$$F_p(\ell) \sim \langle\epsilon(\ell)^{p/3}\rangle\ell^{p/3}. \tag{2}$$

This modified law predicts correction to the linear law $\zeta(p) = p/3$, that are connected to the intermittent nature of the dissipation. For example, a log-normal model for of the dissipation (a suggestion by Landau and Obukhov) implies quadratic corrections for the $\zeta(p)$. Other models have been suggested and lead to different corrections [10, 6, 2]. Intermittency corrections up to $p = 4$ have been measured in a variety of experimental and numerical flows and appear to be robustly consistent from an experiment to another (see e.g. the review of [1]). Corrections for larger values of p are subject to resolution and statistical convergence issues: the larger the scaling exponent, the larger the statistical sampling must be in order to capture the rare events. There is presently no general consensus about the behavior of intermittency corrections at large order. This hinders progress in the understanding of the statistical properties of the energy dissipation. In this Letter, we suggest a new approach to probing intermittency corrections based on the Auto-Regressive Moving-Average (ARMA) modeling of turbulent time series. We introduce a new index Υ that measures the distance from a Kolmogorov-Obukhov model in the ARMA space. Applying our analysis to velocity measurements in a von Kármán swirling flow, we show that this index is proportional to the traditional intermittency correction computed from the structure function and provides the same information, using shorter time series.

INTERMITTENCY AS A DISTANCE IN ARMA SPACE

A stationary time series X_t is said to follow an ARMA(p, q) process if it satisfies the discrete equation:

$$X_t = \sum_{i=1}^{p}\phi_i X_{t-i} + \varepsilon_t + \sum_{j=1}^{q}\theta_j\varepsilon_{t-j}, \tag{3}$$

with $\varepsilon_t \sim WN(0, \sigma^2)$ - where WN stands for white noise - and the polynomials $\phi(z) = 1 - \phi_1 z_{t-1} - \cdots - \phi_p z_{t-p}$ and $\theta(z) = 1 - \theta_1 z_{t-1} - \cdots - \theta_q z_{t-q}$, have no common factors. Notice that the noise term ε_t will be assumed to be a white noise, which is a general condition [4]. We ensure unicity by applying the Box-Jenkis procedure [3]: we choose the lowest p and q such that the residuals of the series filtered by the process ARMA(p, q) are not correlated. To define a suitable distance in the space of ARMA(p, q) models, we use the Bayesian information criterion (BIC), measuring the relative

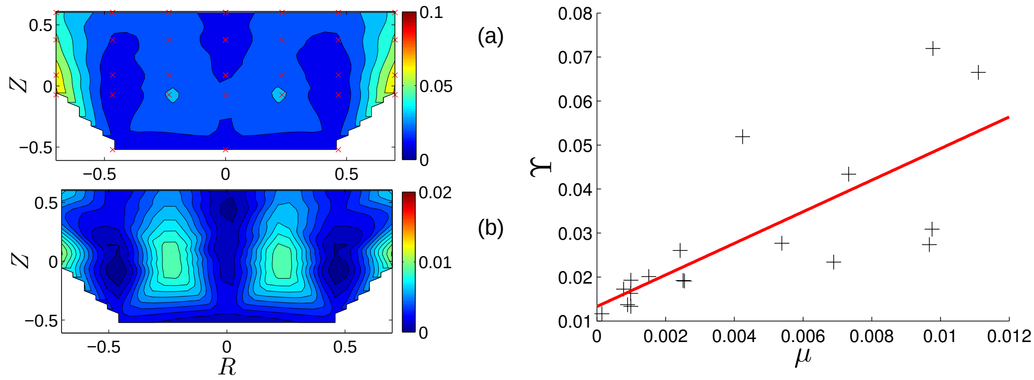

Figure 1. (a) Index Υ vs (b) the intermittency index $\mu = \zeta_2^* - \frac{2}{3}\zeta_3^*$. White crosses show measurement points. (c) scatter plot of Υ vs μ The red (gray) line shows a linear regression of the data.

quality of a statistical model. The normalized distance between the fit ARMA$(p+1,q)$ and the Kolmogorov AR(1) model is then defined as the normalized difference between the $BIC(n,\hat{\sigma}^2,p+1,q)$ and the AR(1) $BIC(n,\hat{\sigma}^2,1,0)$:

$$\Upsilon = 1 - \exp\left\{|BIC(p+1,q) - BIC(1,0)|\right\}/n \qquad 0 \leq \Upsilon \leq 1. \tag{4}$$

The $p+1$ serves to magnify Υ near zero. $\Upsilon \to 0$ if the dataset is well described by an AR(1) model and, $\Upsilon \to 1$ in the opposite case. In the case of velocity increments time series, it measures deviations from the Kolmogorov model.

APPLICATIONS

We apply the index defined in eq. 4 to velocity time series obtained in a von Kármán turbulent swirling flow. The experimental set-up consists of two sets of blades mounted on two counter-rotating co-axial impellers at the top and bottom of a cylindric vessel of diameter $R = 0.1$ m. The operating fluid is water, the rotation frequency of the impellers can reach $F = 15$ Hz, resulting in large Reynolds numbers ($Re = 2\pi F R^2 \nu^{-1} \sim 10^6$). A detailed description of the experiment can be found in [5, 8]. The Laser Doppler Velocimetry (LDV) used to measure the velocity fields which are mapped on a regular sampling time applying a sample-and-hold algorithm. The LDV measurements provide the out-of-plane velocity component V_ϕ into a plane. The LDV time-series are sampled over time-scale of the order of 0.001 s, producing sample size up to 10^6 data on a grid of spatial resolution of the order of 1 cm. Given these resolutions constraints, we compute temporal velocity increments for the LDV data. At each spatial grid location we compare the classical intermittency index μ to Υ , and see how they vary. All the analyses. Since the von Kármán flow is inhomogeneous and anisotropic with large fluctuations [5], we expect that the time and space velocity structure functions depend on the measurement points. Using these ESS scaling exponents to compute the μ index, we may then draw a map of the intermittency and compare it with Υ. This is done in Fig. for an LDV experiment at $Re \sim 10^5$. The spatial patterns look indeed similar. Moreover, the plot of Υ as a function of μ (Fig. -(c)) evidences a linear relation between them with a linear regression coefficient $r \simeq 0.69$. This means that Υ traces the same intermittency characteristics as the time structure functions. See [7] for more details.

References

[1] A Arneodo, C Baudet, F Belin, R Benzi, B Castaing, B Chabaud, R Chavarria, S Ciliberto, R Camussi, F Chilla, et al. Structure functions in turbulence, in various flow configurations, at reynolds number between 30 and 5000, using extended self-similarity. *EPL (Europhysics Letters)*, **34**(6):411, 1996.

[2] Roberto Benzi, Giovanni Paladin, Giorgio Parisi, and Angelo Vulpiani. On the multifractal nature of fully developed turbulence and chaotic systems. *Journal of Physics A: Mathematical and General*, **17**(18):3521, 1984.

[3] G. E. Box and G. M. Jenkins. *Time Series Analysis: Forecasting and Control.* Holden-D. iv, 1970.

[4] P. J. Brockwell and R. A. Davis. *Time Series: Theory and Methods, second edition*. Springer, 1990.

[5] Pierre-Philippe Cortet, Pantxo Diribarne, Romain Monchaux, Arnaud Chiffaudel, François Daviaud, and Bérengère Dubrulle. Normalized kinetic energy as a hydrodynamical global quantity for inhomogeneous anisotropic turbulence. *Physics of Fluids*, **21**:025104, 2009.

[6] Bérengère Dubrulle. Intermittency in fully developed turbulence: Log-poisson statistics and generalized scale covariance. *Physical review letters*, **73**(7):959, 1994.

[7] Davide Faranda, Bérengère Dubrulle, François Daviaud, and Flavio Maria Emanuele Pons. Probing turbulence intermittency via autoregressive moving-average models. *Physical Review E*, **90**(6):061001, 2014.

[8] Davide Faranda, Flavio Maria Emanuele Pons, Bérèngere Dubrulle, François Daviaud, Brice Saint-Michel, Éric Herbert, and Pierre-Philippe Cortet. Modelling and analysis of turbulent datasets using arma processes. *Physics of fluid*, **26 (10)**, 2014.

[9] Andrey Nikolaevich Kolmogorov. A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high reynolds number. *Journal of Fluid Mechanics*, **13**(01):82–85, 1962.

[10] Zhen-Su She and Emmanuel Leveque. Universal scaling laws in fully developed turbulence. *Physical review letters*, **72**(3):336, 1994.