SPACE-TIME RECONSTRUCTION OF FINELY RESOLVED VELOCITIES OF TURBULENT FLOWS FROM LOW RESOLUTION MEASUREMENTS

Linh Van Nguyen[1] , Jean-Philippe Laval [1] & Pierre Chainais [2]

[1] *Laboratoire de Mecanique de Lille (LML), CNRS-UMR 8107, F-59650 Villeneuve d'Ascq, France*

[2] *Ecole Centrale Lille, CRIStAL-CNRS UMR 9189, INRIA SequeL, F-59650 Villeneuve d'Ascq, France*

Abstract A model is developed to reconstruct the High-Temporal-High-Spatial (HTHS) resolution velocities of a wall bounded turbulent flow combining two sources of measured data: the Low-Temporal-High-Spatial (LTHS) and the High-Temporal-Low-Spatial (HTLS) resolution measurements. Direct Numerical Simulation (DNS) database of a turbulent channel flow is used. LTHS and HTLS are subsampled (in time or space) from the fully resolved data. A fusion model is constructed in Bayesian framework using a Maximum A Posteriori (MAP) estimation. By maximizing the posterior probability of desired HTHS conditional on LTHS and HTLS, the model estimates the most probable fields knowing the measurements. Reconstructed velocities are compared to the reference to qualify the fusion model and to those by conventional Linear Stochastic Estimation (LSE) and interpolation methods to demonstrate its robustness.

INTRODUCTION

Turbulence is a multi-scale phenomenon, where a very wide range of scales co-exist and interact within the same flow. Therefore, although governed by Navier-Stokes equations in most practical problems, turbulence is still very hard to predict and model. In order to understand its detailed physics, space-time resolved information is desired. Unfortunately, none of the current facilities, even in the context of academic research, is capable to provide this information in a sufficiently wide spatial domain and for diverse flow conditions. Despite the advancement of computational resources, DNS is limited to the flows with low to moderate Reynolds numbers or those in simple geometries. Measurement techniques such as PIV and HWA are not able to measure space-time resolved velocities. Although high repetition tomographic PIV is progressing, it is still limited to small volumes and to low speed flows. HWAs give temporal resolved measurements, but the combination of many HWs is not straightforward and remains intrusive.

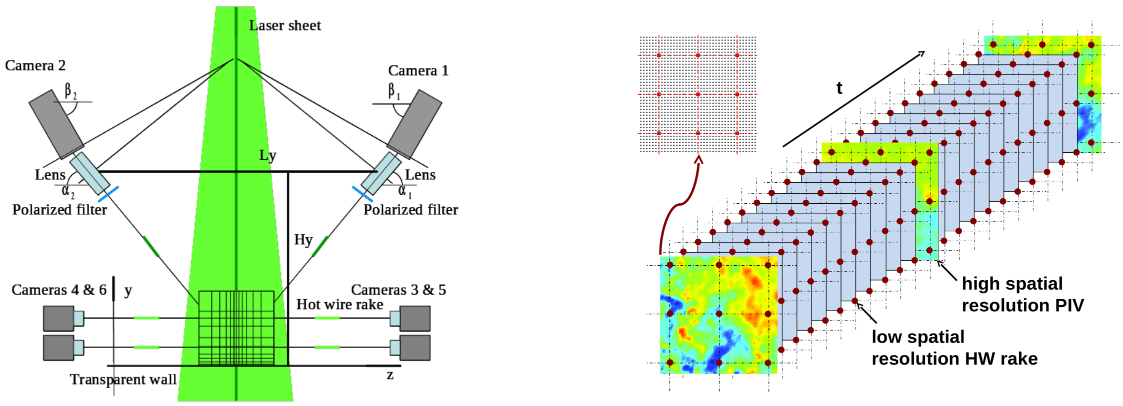

Figure 1. Sketch of the problem: (left) Experimental setup of TBL measurement with 2D stereo PIV and a rake of 143 HWA probes [1]; (right) setup of the inverse problem, with the two sources of measurements: the LTHS (color images) and a coarse grid of HTLS (red dots among black ones of LTHS). The inverse problem of HTHS data reconstruction is to fill in the data-cube.

Taking into account above limitations, an experiment was performed in LML (figure 1) to measure the two datasets of a full Turbulent Boundary Layer (TBL) at high Reynolds numbers, the PIV measurements (LTHS) and a coarse grid of 143 HWA probes (HTLS) [1]. This work aims to develop a Bayesian fusion model to reconstruct the fully resolved velocities that are not directly given by the measurements. These fields are compromise estimates of the two measured data sources. The final objective is to extract dynamical behaviors of coherent structures in TBL flows using the estimated velocities. Moreover, the present work potentially provides a compression algorithm for turbulence data, allowing to record sparse data and restore maximum information of turbulence physics given a certain amount of kinetic energy losses.

RECONSTRUCTION METHOD AND RESULTS

Methods: Assuming turbulent velocities follow Gaussian distributions, a fusion model is developed to combine measured data. It is extended from a model for image processing problems [2]. Let $\boldsymbol{x}$, $\boldsymbol{y}$ and z denote LTHS, HTLS and HTHS respectively. The *posterior probability* of z conditional on $\boldsymbol{x}$ and $\boldsymbol{y}$ is $p(z|\boldsymbol{x},\boldsymbol{y})$. This is the conditional probability of unknown HTHS knowing the measurements of LTHS and HTLS. The fusion model uses a MAP estimation to maximize

$p(z|\boldsymbol{x}, \boldsymbol{y})$ with respect to z, i.e. $\hat{z} = \text{argmax}\{p(z|\boldsymbol{x}, \boldsymbol{y})\}$. The estimates $\hat{z}$ are the most probable fields given $\boldsymbol{x}$ and $\boldsymbol{y}$. The solution of this optimization problem for one point in space-time $(\vec{s}, t)$ takes the form of a weighted average:

$$\hat{z}(\vec{s}, t) = \frac{\sigma_s^2(\vec{s})}{\sigma_s^2(\vec{s}) + \sigma_t^2(t)} \mathbb{I}_t \boldsymbol{x}(\vec{s}, t) + \frac{\sigma_t^2(t)}{\sigma_s^2(\vec{s}) + \sigma_t^2(t)} \mathbb{I}_s \boldsymbol{y}(\vec{s}, t) \tag{1}$$

Notations $\mathbb{I}_t$ and $\mathbb{I}_s$ represent the time and space interpolators such that $\mathbb{I}_t\boldsymbol{x}$ and $\mathbb{I}_s\boldsymbol{y}$ have the same space-time resolution as z. The variances σ_s^2 and σ_t^2 are Mean Square Errors (MSE) of interpolated velocities and are learned from the measurements.

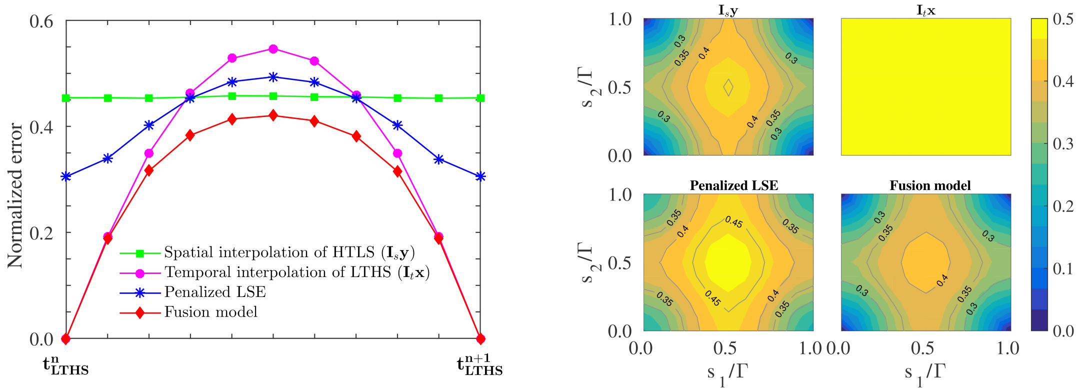

Figure 2. Averaged normalized errors $\epsilon = \sqrt{\sum_i (z_i - \hat{z}_i)} / \sqrt{\sum_i z_i}$ between reference and reconstructed streamwise velocities as: (left) function of time between LTHS snapshots at the most-difficult-to-estimate spatial location (furthest from the four nearby HTLS); (right) function in space (four HTLS at the corners of spacing Γ) at the most-difficult-to-estimate instant (equally far from nearest LTHS).

Database: The DNS data of a turbulent channel flow at Reynolds number $Re_\tau = 550$ and domain size (normalized by channel height) $2\pi \times \pi \times 2$ is used to develop the model. HTHS of fluctuating streamwise velocities z are extracted in a 2D plane normal to the flow, including 10000 snapshots at resolution of 288×257 and frequency of 40 Hz. This dataset is divided into two parts: the training data of measurements $\boldsymbol{x}$ and $\boldsymbol{y}$ to build the fusion model, and the testing data of original DNS to compute the reconstruction errors. Measurements $\boldsymbol{x}$ and $\boldsymbol{y}$ are subsampled at the ratios of 10 in space or time from DNS data. While $\boldsymbol{x}$ is at DNS resolution of 288×257 and frequency 4 Hz, $\boldsymbol{y}$ is resolved at 40 Hz but subsampled in a coarse grid of 26×29 points. Errors of estimated $\hat{z}$ is computed using reference z from DNS.
Conventional methods: Standard techniques such as spline interpolation and regression are widely used for these reconstruction problems. The interpolation technique reconstructs velocities of HTHS from LTHS or HTLS independently, i.e. $\boldsymbol{x} \longmapsto \hat{z} = \mathbb{I}_t\boldsymbol{x}$ or $\boldsymbol{y} \longmapsto \hat{z} = \mathbb{I}_s\boldsymbol{y}$. Regression methods such as LSE aim to estimate $\hat{z}$ as a linear combination of measurements via a set of coefficients, i.e $\boldsymbol{x} \longmapsto \hat{z} = B_1(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{x}$ or $\boldsymbol{y} \longmapsto \hat{z} = B_2(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{y}$. Matrices B_1 and B_2 are computed from $\boldsymbol{x}$ and $\boldsymbol{y}$ by solving systems of linear equations (to minimize MSE of reconstructed fields) using Tikonov- (or L2-) regularization or Proper Orthogonal Decomposition (POD).
Results: Reconstructed fields by the proposed and other models are compared to the original DNS data. The fusion model gives better estimates at all positions in space and time. Figure 2 shows the errors between reconstructed and original velocities at the most difficult positions in space (as a function of all instants between two adjacent LTHS) and in time (as a function of space restricted by four HTLS at the corners). Even at these positions, the fusion model shows better performances. This is because spline techniques interpolate resolved velocities either from HTLS or LTHS, therefore losing the information from the other source. LSE regression model learns the matrices B_1 and B_2 from all measurements. However, these matrices tend to average correlations among them. Also, estimated B_1 and B_2 are then performed on either HTLS or LTHS data, implying a loss of local correlation from the missing source. The fusion model overcomes these limitations by capturing all space-time correlations simultaneously. Acting as a weighted average, the estimation (1) uses $\sigma_s^2(\vec{s})$ and $\sigma_t^2(t)$ as the statistical measures of flow physics, and $\boldsymbol{x}$ and $\boldsymbol{y}$ as specific information about the observed flow. Close to measurements, the model impose the reconstruction to follow; otherwise, compromise estimates are proposed.
Conclusions: As an extension of the MAP model in image processing, the fusion model is successfully applied to turbulent flow reconstruction and show better performance compared to other standard techniques.

References

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