TURBULENT PLANE COUETTE FLOW WITH WALL-TRANSPIRATION

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Abstract In the present abstract, DNS results obtained for turbulent plane Couette flow with wall-normal transpiration velocity v_0 are presented. Important equations valid in such a flow are derived, describing the total shear stress and the relation between the friction velocities at the lower and upper wall. These expressions are of importance, as there are neither experimental nor DNS data to compare with. Equally important, we derive a center region and a viscous sublayer velocity scaling for the suction wall, which were both validated using the DNS data.

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The present work deals with fully developed turbulent plane Couette flow with wall-normal transpiration velocity, i.e. uniform blowing and suction at the lower and upper wall, respectively. Fig. 1 gives a schematic impression of the flow investigated herein compared to classical Couette flow. Couette flow is a fundamental flow featuring distinguishing characteristics which make it a commonly studied wall-bounded canonical flow, for instance, see [1] and references therein. However, to the authors' best knowledge neither experimental nor numerical studies have been performed on Couette flow with permeable boundary conditions (BC) yet.

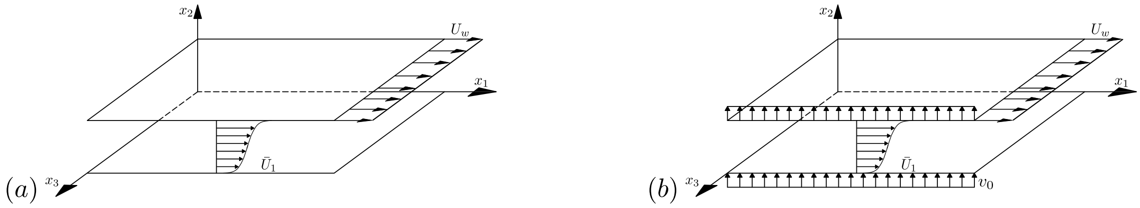

Figure 1. Schematic sketch of classical plane Couette flow (a) compared to plane Couette flow with wall-normal transpiration velocity (b) discussed in the present abstract.

A DNS code which originally was developed at the School of Aeronautics, Technical University of Madrid, see [3], is used for the simulations presented in this abstract. The computational domain is a $(L_x \times L_y \times L_z) = (8\pi h \times 2h \times 3\pi h)$ box, where h is the channel half-width. This box size is similar to the ones used in Poiseuille flows, since for the case with wall-transpiration it can be reduced drastically compared to the case with classical, i.e. non-permeable, BC. The large structures which appear in classical Couette flow requiring adequate large box sizes (see e.g. [4]) seem to vanish in the case of Couette flow with wall-transpiration.
One of the distinguishing features of turbulent plane Couette flow is the fact that the total shear stress is equal to one across the whole channel height. However, if a constant wall-normal transpiration velocity v_0 is imposed as a BC at the walls, the mean momentum equation extends to

$$v_0 \frac{\mathrm{d}\langle U\rangle}{\mathrm{d}y} = \nu \frac{\mathrm{d}^2\langle U\rangle}{\mathrm{d}y^2} - \frac{\mathrm{d}\langle uv\rangle}{\mathrm{d}y}. \tag{1}$$

From this, the key finding is that the friction velocity at the lower and upper wall differ due to the effects of transpiration: Blowing on the lower wall leads to a lower friction velocity, while suction on the upper wall results in a higher u_τ. Integration of (1), employing appropriate BC and normalization by the friction velocity at the lower wall, namely u_{τ_0}, and the kinematic viscosity ν yields a new equation for the total shear stress,

$$1 = \frac{\mathrm{d}\langle U\rangle}{\mathrm{d}y}^{+0} - \langle uv\rangle^{+0} - v_0^{+0}\,\langle U\rangle^{+0}. \tag{2}$$

Another useful equation, namely

$$Re_{v_0} Re_{U_w} = Re_{\tau 2h}^2 - Re_{\tau 0}^2, \tag{3}$$

can be derived from a global momentum equation, quantifying the relation between the independent parameters v_0, U_w (velocity of the upper wall), ν, h and the wall shear stresses at the lower and upper wall, τ_{w_0} and $\tau_{w_{2h}}$, respectively. Normalization yields Eq. (3), where the Reynolds numbers are defined as $Re_{v_0} = v_0 h/\nu$, $Re_{U_w} = U_w h/\nu$, $Re_{\tau 2h} = u_{\tau_{2h}} h/\nu$ and $Re_{\tau 0} = u_{\tau_0} h/\nu$. Eqs. (2) and (3) provide physical information which can be used as a tool to assure a statistically steady flow.

VISCOUS SUBLAYER VELOCITY SCALING AT THE SUCTION WALL

With increasing v_0, at the suction wall, the flow tends to a local relaminarization. Analogously to [2], where Poiseuille flow with wall transpiration has been investigated, a velocity scaling in the viscous sublayer at the suction wall can be derived by integrating the momentum equation (1) in the limit of $\langle uv \rangle \longrightarrow 0$. Additionally, a new coordinate system $(x_s, y_s) = (-x, 2h - y)$ is introduced, with y_s being the new wall-normal coordinate at the suction wall. The coordinate system is fixed at the suction wall and thus the corresponding velocity is $\langle U_s \rangle = U_w - \langle U \rangle$. Applying such assumptions, we obtain

$$\overline{U}_s^+ = \frac{\langle U_s \rangle v_0}{u_{\tau_s}^2} = \frac{U_w v_0}{u_{\tau_s}^2} + \exp\left(-2Re_{v_0}\right) - \exp\left(-y_s^+\right), \tag{4}$$

using the wall-based scaling derived in [5] (see also [2]), where u_{τ_s} is the friction velocity at the suction wall and $y_s^+ = v_0 y_s/\nu$. In the limit of $Re_{v_0} \longrightarrow \infty$ while at the same time $y_s^+ = \mathcal{O}(1)$, the scaling of $\langle U_s \rangle$ simplifies to

$$\overline{U}_s^+ = 1 - \exp\left(-y_s^+\right). \tag{5}$$

As can be seen in Fig. 2a, the DNS results presented here agree well with Eq. (5), i.e. results converge to the exponential scaling law as v_0 increases.

LOGARITHMIC VELOCITY SCALING IN THE CHANNEL CENTER

Another scaling law valid in the center of the channel was derived in [2], namely

$$\frac{\langle U \rangle - U_b}{u_\tau} = \frac{1}{\gamma} \ln\left(\frac{y}{h}\right), \tag{6}$$

where U_b is the bulk velocity, γ is a constant and $u_\tau = \sqrt{(u_{\tau 2h}^2 + u_{\tau 0}^2)/2}$ is the global friction velocity. It was shown in [2], that the logarithmic scaling law (6) is true for Poiseuille flow with wall-transpiration. Likewise, good accordance was found for the presently investigated case of Couette flow with wall-transpiration, see Fig. 2b.

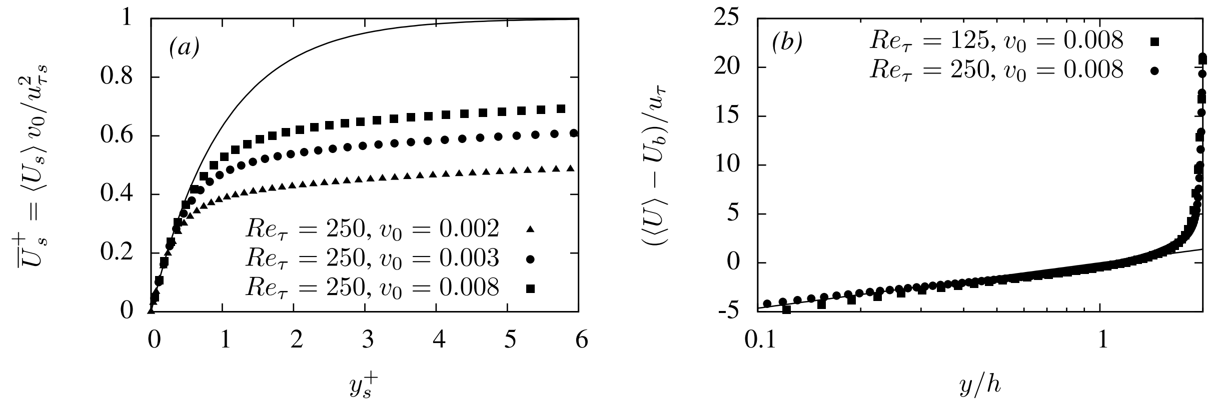

Figure 2. Scaling laws valid for Couette flow with wall-transpiration. *(a)*: Mean velocity profiles at $Re_\tau = 250$ and different transpiration velocities v_0 (symbols) and scaling law (5) at the suction wall (solid line). *(b)*: Logarithmic scaling law (6) with $\gamma \approx 0.5$ (solid line) and results of cases with $v_0 = 0.008$ and different Re_τ (symbols).

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References

[1] V Avsarkisov, S Hoyas, M Oberlack, and JP García-Galache. Turbulent plane couette flow at moderately high reynolds number. *Journal of Fluid Mechanics*, **751**:R1, 2014.

[2] V Avsarkisov, M Oberlack, and S Hoyas. New scaling laws for turbulent poiseuille flow with wall transpiration. *Journal of Fluid Mechanics*, **746**:99–122, 2014.

[3] S Hoyas and J Jiménez. Scaling of the velocity fluctuations in turbulent channels up to reτ= 2003. *Physics of Fluids*, **18**(1):011702, 2006.

[4] J Komminaho, A Lundbladh, and A V Johansson. Very large structures in plane turbulent couette flow. *Journal of Fluid Mechanics*, **320**:259–285, 1996.

[5] H Tennekes. Similarity laws for turbulent boundary layers with suction or injection. *Journal of Fluid Mechanics*, **21**(04):689–703, 1965.