INFLUENCE OF SMALL-SCALE TURBULENCE ON SPATIAL DISTRIBUTION OF CLOUD-LIKE PARTICLES

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Abstract Influence of small-scale turbulence on cloud droplets spatial distribution was analyzed by examination of motion of inertial droplets in a simple model of a vortex tube aligned at arbitrary angle to gravity force. Both analytical calculations and numerical simulations demonstrated characteristic features of the motion such as equillibrium points, limit cycles and stationary orbits, for which conditions of existence were calculated. Simulations of motion of polydispersed droplets illustrate sorting effects of a vortex tube.

INTRODUCTION

Many studies attribute the evolution of droplet size distribution in warm convective clouds to enhancement of collision-coalescence by turbulence (see e.g. reviews by [1], [2], [3]). One of the influences of turbulence on droplets is preferenial concentration - uneven distribution of droplets in space. In order to present explicitely one of possible mechanisms of preferential concentration we analyze motion of heavy inertial droplets in simple model of a vortex tube. Vortex tubes are small coherent structures characteristic for high Reynolds number turbulence ocurring in clouds. Former research of such effects by Hill [4] and by Markowicz [5] was performed in two dimensions and was limited to horizontally oriented vortex tubes only. Herein we analyze three dimensional motion of droplets in vortices which are parallel or oblique to the direction of gravity. Similar analysis with a use of Burgers vortex as a model of a vortex tube was done by Marcu et al. in [6].

MODEL DESCRIPTION

Line vortex is three dimensional flow structure of constant circulation and singular vorticity concentrated on a straight line. We used constant velocity field generated by line vortex of circulation $\Gamma = 2\pi L_w$ with stretching of strenght γ as a model of vortex tube $\vec{v_a} = -\frac{\gamma}{2} r\hat{e}_r + \frac{L_w}{r}\hat{e}_\phi + \gamma z\hat{e}_z$. We arranged our vortex model to cover all possible orientations with respect to gravity direction by introducing an angle $\theta \in [0, \pi]$ between gravity vector and vortex axis. Numerical simulations were done for visualization purposes with parameters corresponding to cloud/water droplets in an airflow (vortex parameters chosen: $\gamma = 0.5\ \frac{1}{s}$, $L_w = 2.5 \cdot 10^{-4}\ \frac{m^2}{s}$, alignment angle $\theta = 0.45\pi$).

We assumed that droplet is a point particle and its motion in fluid is determined by viscosity and gravity forces only. No other hydrodynamical forces and no interaction with other droplets were included. Stokes equation with gravity was used as droplet equation of motion. In a fluid flow with velocity field v_a, for a droplet of mass m in a position $\vec{r}$ with inertial response time τ and under influence of gravity $\vec{g}$ this equation is expressed by:

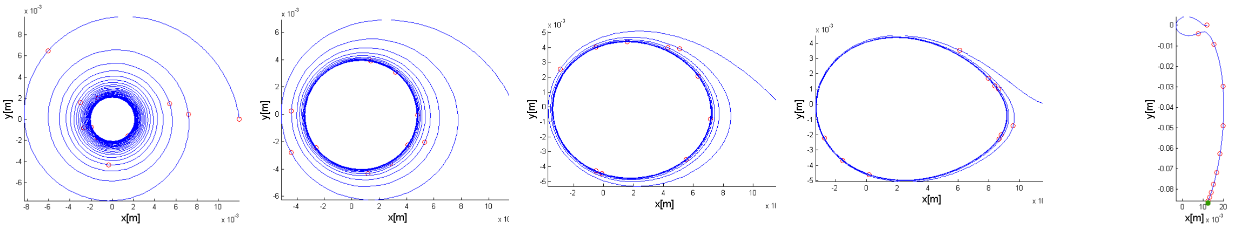
$$m\ddot{\vec{r}} = \frac{1}{\tau} m(\vec{v}_a - \dot{\vec{r}}) + m\vec{g}.$$

After nondimensionalizing the equations of motion in a plane perpendicular to the vortex axis (here in (r, ϕ) coordinates) separate from motion along Z axis and depend on three nonsimensional parameters K_1, K_2 and K_3 depending on all droplet/vortex parameters:

$$\begin{cases} \ddot{r} - r\dot{\phi}^2 = -(\frac{K_1}{2} r + \dot{r} + K_3 sin(\phi)) \\ 2\dot{r}\dot{\phi} + r\ddot{\phi} = \frac{1}{r} - r\dot{\phi} - K_3 cos(\phi)) \\ \ddot{z} = K_1 z - \dot{z} - K_2 \end{cases}.$$

RESULTS

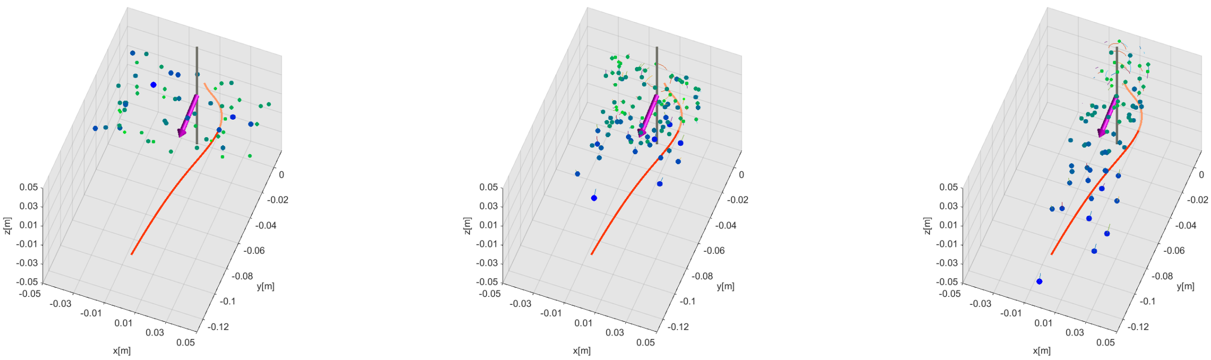
Direction of motion along Z axis for droplet starting with initial conditions $z(0) = z_0,\ \dot{z}(0) = 0$ is determined only by the initial position of droplet z_0. If $z_0 = z_{0b}$ ($z_{0b} \propto \gamma^{-1} R^2 \cos\theta > 0$) the droplet stays in unstable steady position in respect to this motion. If $z_0 > z_{0b}$ or $z_0 < z_{0b}$ droplet moves nearly exponentially up or down respectively along the vortex axis. Motion in a plane perpendicular to the vortex axis shows strong qualitative dependence on angle θ. In the case of "vertical vortex" ($\theta = 0$) there is only one kind of solution for motion in plane (r, ϕ): every droplet has its circular stable, periodic orbit on which radial viscous force and centrifugal force equalize. Radius of this orbit is $r_{orb} = \sqrt[4]{\frac{2\pi}{\gamma}(\frac{\Gamma}{2\pi})^2}$. Stability of the orbit guarantees that trajectories of all the droplets spirals into it in finite time. In the case of "oblique vortex" (with $\theta \neq 0$) gravity influence destroys axial symmetry of motion in a plane perpendicular to the vortex axis so solution of equations in a form of round, stable, periodic orbit does not exist for any droplet. It results however in possibility of apperance of equillibrium points (in the IV quadrant of the plane when described with (x,y) coordinates). Positions of these points are $r_{st\pm} = \sqrt{2}\frac{K_3}{K_1}\sqrt{1 \pm \sqrt{1-\frac{K_1}{K_3^2}}^2}$, $\phi_{st\pm} = -arcsin(\frac{1}{\sqrt{2}}\sqrt{1 \pm \sqrt{1-\frac{K_1}{K_3^2}}^2})$. They may exist under the condition $K_3^2 \geq K_1$, which is equivalent with the statement that for given vortex of γ and L_w equillibrium point may exist only for droplets of radii above certain boundary radius value. Numerical simulations show also apperance of noncircular limit cycle under certain vortex/droplet conditions. It can be a unique stable solution for a given droplet or compete with the stable equillibrium point. Generally the result of leaning the line vortex with respect to gravity leads to droplet two dimensional motion in which it approaches one of two types of attractors: either a stable limit cycle or a stable equillibrium point. This conclusion is visualized in the Figure 1.



(a) $R = 2.5\mu m$ (b) $R = 9\mu m$ (c) $R = 12\mu m$ (d) $R = 13\mu m$ (e) $R = 13.5\mu m$

Figure 1. Trajectories of droplets od various radiuses R for $\theta = 0.45\pi$. In this vortex boundary radius value is $9,56\mu m$. Droplets in figures (a)-(b) are attracted by unique stable solutions in form of differently shaped limit cycles. Droplets in figures (c)-(d) are as well attracted by limit cycles regardless of having also equillibrium positions. Droplet in figure (e) is attracted by its equillibrium point.

The consequences of the specific types of droplet motion for their spatial distribution in three dimensions are concentration of same size droplets in some regions and separation of different size droplets. This effect is presented in the simulation frames for polydispersed droplets in Figure 2.



(a) t=0 s (b) t=2 s (c) t=4 s

Figure 2. Frames from simulation for polydispersed group of droplets from the range $R \in [1, 20\mu m]$ (from green to blue). Purple vector marks direction of gravity, grey straight line is vortex axis, red and orange lines are collections of analytically calculated equillibrium point positions (r_+ and r_- respectively) existing for some droplets from the range used.

References

[1] Devenish, B. J. et. al., Review Article. Droplet growth in warm turbulent clouds. *Q. J. R. Meteorol. Soc.* **138**, 1401-1429, July 2012 B.
[2] Shaw, R. A., Particle-turbulence interactions in atmospheric clouds. *Annu. Rev. Fluid Mech.* **35**, 183-227, 2003.
[3] Grabowski, W.W., Wang, L.-P. Growth of Cloud Droplets in Turbulent Environment. *Annu. Rev. Fluid Mech.***45**, 293-324, 2013.
[4] R. J. Hill. Geometric collision rates and trajectories of cloud droplets falling into a Burgers vortex *Phys. Fluids* **17**: 037103, 2005.
[5] K. P. Markowicz, K. Bajer and S. P. Malinowski. Influence of the small-scale turbulence structure on the concentration of cloud droplets. *Proc. of 13th Conf. on Clouds and Precip., IAMAP*: 159-162, 2000.
[6] B. Marcu, E. Meiburg and P. K. Newton. Dynamics of heavy particles in a Burgers vortex *Phys. Fluids* **7**: 400-410, 1995.