OBSERVATION OF PREDATOR-PREY DYNAMICS AND THE UNIVERSALITY CLASS IN TRANSITIONAL PIPE TURBULENCE

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Abstract Near the onset to turbulence in pipes, around Re $\sim 1700 - 2000$, turbulent puffs decay either directly or, at higher Reynolds numbers through splitting, with characteristic time-scales that exhibit a super-exponential dependence on Reynolds number [3, 1, 7]. The goal of our work [5] is to understand the phenomenology of this transition in terms of standard phase transition concepts, and to calculate the universality class from first principles. Using direct numerical simulations (DNS) of transitional pipe flow, we show that a collective mode, a so-called zonal flow emerges at large scales, activated by anisotropic turbulent fluctuations through an inverse cascade of energy. This zonal flow imposes a shear on the turbulent fluctuations that tends to suppress their anisotropy, leading to stochastic predator-prey dynamics. The effective stochastic theory for the predator-prey modes identified in the DNS reproduces the super-exponential lifetime statistics and phenomenology of pipe flow experiments, correctly predicts the phase diagram of transitional turbulence, and can be mapped exactly to the field theory of directed percolation.

BACKGROUND

Nearly 130 years after Osborn Reynolds reported that the transition to turbulence occurs through the appearance and decay of turbulent puffs, major advances in experimental technique and computation have led to the prospect that this transition can be completely understood and characterized (for a recent review, see [7]). The definitive measurement of the decay lifetime, τ^d, of puffs as a function of Re [3] showed that $\ln \ln \tau^d \propto$ Re. For Reynolds numbers based upon pipe diameter D of around 2300, the puffs become unstable through a new dynamical processes in which the leading edge breaks away and nucleates the formation of a new puff downstream. The puff-splitting time scale also decays super-exponentially with increasing Re. Our goal was to develop a theory for the transition following standard approaches from phase transitions [2]. Such a theory would require the identification of an effective theory at a coarse-grained level of description, based upon the appropriate long-wavelength collective modes, as well as a detailed account of their interactions and fluctuations. Since it almost impossible to derive effective theories from microscopic models, in this case the Navier-Stokes equations, we instead used DNS to seek and identify candidate collective modes. This talk is a summary of a recent preprint [5].

OBSERVATION OF PREDATOR-PREY OSCILLATIONS IN SIMULATIONS OF TRANSITIONAL PIPE FLOW

We used the open-source code "Open Pipe Flow" [8], kindly provided to us by A. Willis, and discovered a collective mode in the form of an azimuthally-symmetric zonal flow that is generated by the anisotropy of the turbulent fluctuations. The velocity field is sketched in Fig 1 (A). The energy of turbulent modes and the zonal flow are shown as a function of time in Fig 1 (B), exhibiting noisy oscillations that are $\pi/2$ out of phase and characteristic of stochastic predator-prey cycles in ecology. The zonal flow is activated by the energy of the turbulent fluctuations, and so is the "predator". In Fig 1 (C) we plot the radial dependence of the mean zonal flow velocity in the azimuthal direction, $\overline{v}_\theta$, together with the radial gradient of the Reynolds stress tensor, showing that the latter drives the former.

PIPE FLOW PHENOMENOLOGY PREDICTED BY PREDATOR-PREY MODEL

The stochastic predator-prey behavior can be described by an individual-level model formulated as a master equation. We used this effective model to calculate the behavior of puffs as a function of prey birth rate, b, the ecosystem parameter analogous to Re. In Fig 2 (A) we show the phase diagram of pipe turbulence compared with that of the ecosystem model. The ecosystem model exhibits puff decay and splitting just as in the turbulence experiments. Fig 2 (B) shows the world line of splitting puffs, a result that strongly resembles experimental and DNS results [1]. Lastly in Fig 2 (C) we show the lifetime of turbulent puffs to decay (left) and splitting (right), exhibiting a clear super-exponential behavior well fit by a double exponential dependence on prey birth rate, as shown in the inset.

DISCUSSION

The logic of our work is that we used DNS to identify the important collective modes at the onset of turbulence, and then wrote down the simplest minimal stochastic model to account for the observations. This model *predicts* without using the Navier-Stokes equations the puff lifetime and splitting behavior observed in experiment. Moreover, the stochastic model can be transformed using standard statistical mechanics techniques into a field theory known to be in the universality class of directed percolation, as conjectured originally by Pomeau [4] and shown to reproduce much of the phenomenology of transitional turbulence by later simulations [6]. These results provide an unbroken link between the equations of fluid dynamics and the directed percolation universality class [4, 6]. Our approach is a precise parallel to the way in which phase transitions are understood in condensed matter physics, and shows that concepts of universality and effective theories are applicable to the laminar-turbulence transition.

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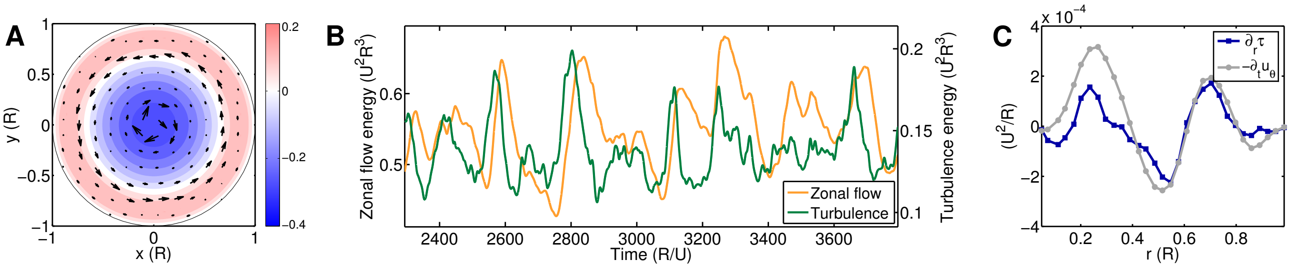

Figure 1. Predator-prey oscillations for pipe of radius R in the transitional Reynolds number range. (A) Velocity field configuration of the zonal flow mode $\vec{\overline{u}}$. The color bar indicates the value of $\overline{u}_z$. (B) Energy vs. time for the zonal flow (orange) and turbulent modes (green). (C) Snapshot of the Reynolds stress gradient, $\partial_r \tau$, and zonal flow time-derivative as functions of radial coordinate r.

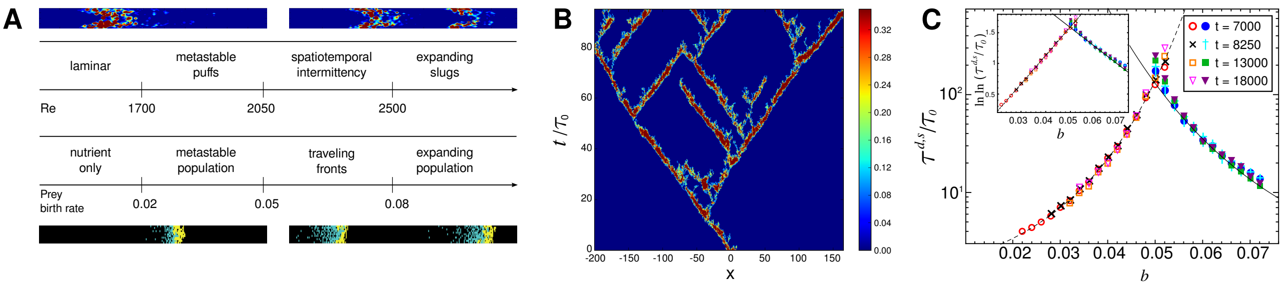

Figure 2. Stochastic predator-prey model reproduces the phenomenology of transitional pipe turbulence. (A) Schematic phase diagram for transitional pipe turbulence as a function of Reynolds number compared with the phase diagram for predator-prey dynamics as a function of prey birth rate. Above each phase is shown a typical flow or predator-prey configuration, indicating the similarity between the turbulent pipe and ecosystem dynamics. (B) World line of clusters of prey splitting to form predator-prey traveling waves. The color measures the local density of prey, corresponding to intensity of turbulence in pipe flow. (C) Log lifetime of prey cluster, τ^d, and splitting time, τ^s, as a function of prey birth rate, b. The upward curvature signifies super-exponential behavior. Inset: Double log lifetime vs prey birth rate, showing the fit to the following functional forms: the dashed curve is given by $\tau^d/\tau_0 = \exp(\exp(46.539b - 0.731))$, and the solid curve is given by $\tau^s/\tau_0 = \exp(\exp(-31.148b - 3.141))$

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