QUASI-STEADY QUASI-HOMOGENEOUS (QSQH) DESCRIPTION OF THE RELATIONSHIP BETWEEN LARGE-SCALE AND SMALL-SCALE MOTIONS IN NEAR-WALL TURBULENCE

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Abstract The validity of the recently proposed hypothesis that the influence of large-scale motions on the near-wall turbulence is quasi-steady is investigated by quantitative comparisons with the data obtained by direct numerical simulations of a turbulent channel flow. Large-scale motions are filtered by a Fourier cut-off filter multi-objectively optimised to increase the correlation between large-scale motions near and away from the wall while decreasing the correlation between small-scale motions. The quasi-steady hypothesis is found to be approximate. It is also found that adding non-linear terms into calculations will improve the accuracy of the prediction.

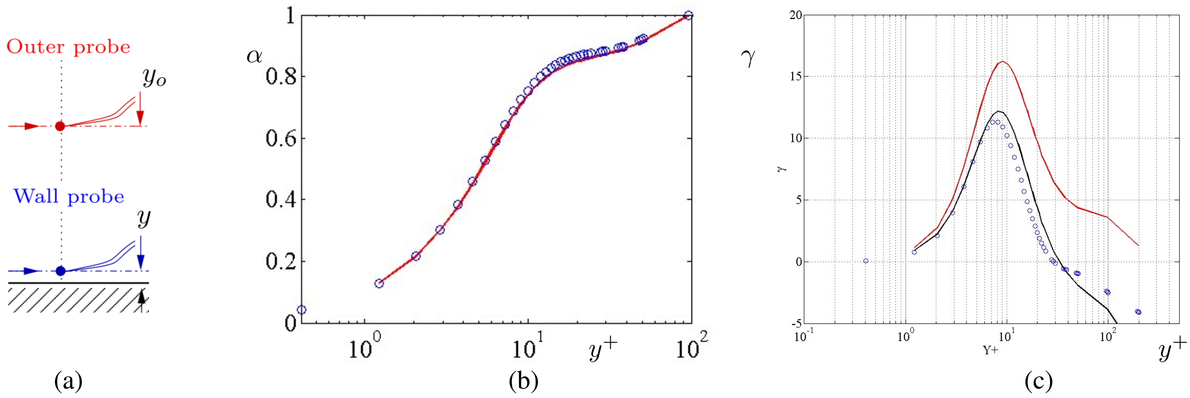


Figure 1. Probe setup (a); coefficients $\alpha_u(y^+)$ (b) and $\gamma(y^+)$ (c) as given by the theory (line) and numerical simulation (points).

Experiments and numerical simulations of turbulent flows are done at moderate Reynolds numbers Re because at high Re simulations would require very large computational resources, while in experiments the measurements near the wall would require probes of very small dimensions and very high frequency range. Extrapolation of the results of such experiments and calculations to the typical high-Re flight regime was justified in the past by universality of near-wall turbulence, in the sense that the statistical properties of the function $u^*(t^+, x^+, y^+, z^+)$, defined from the formula

$$u = \bar{u}_\tau u^*(t^+, x^+, y^+, z^+), \tag{1}$$

are independent of Re. Here, superscript + refers to quantities in wall units, $t^+ = t\bar{u}_\tau^2/\nu$, $x^+ = x\bar{u}_\tau/\nu$, $y^+ = y\bar{u}_\tau/\nu$, $z^+ = z\bar{u}_\tau/\nu$, u is the velocity, $\bar{u}_\tau = \sqrt{\bar{\tau}/\rho}$, $\bar{\tau}$ is the mean skin friction, and other quantities also have the usual meaning. However, distinctive alterations to near-wall turbulence by outer large-scale structures have been observed and confirmed in several recent experimental and numerical studies, see for example [4] and references therein. Instead of (1), Marusic *et al.* [4] proposed the relation

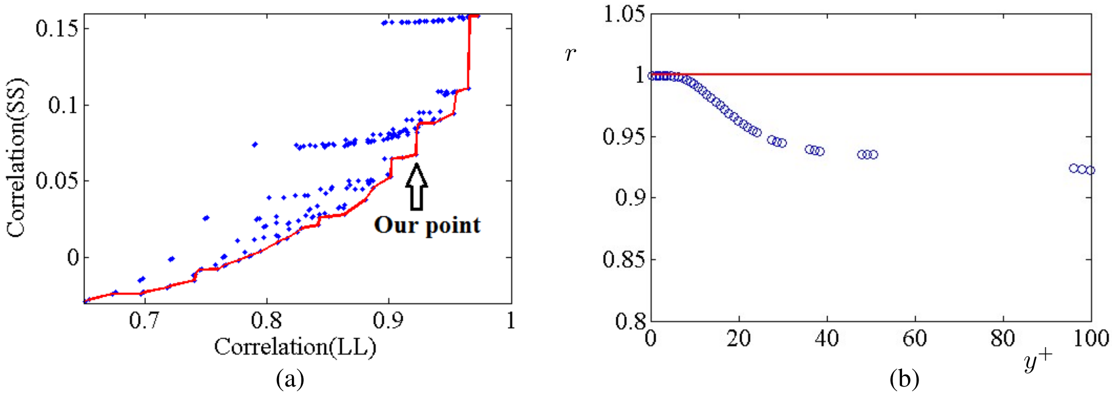
$$u'^+(y^+) = \alpha_u(y^+)u'_{OL} + \left(1 + \beta_u(y^+)u'_{OL}\right) u^*_{\text{MHM}}(t^+, x^+, y^+, z^+), \tag{2}$$

where $\alpha_u(y^+)$ and $\beta_u(y^+)$ are universal Re-independent functions found empirically, prime denotes fluctuations, and the subscript OL marks the large-scale filtered velocity measured by a second probe located further away from the wall, as illustrated by Fig. 1(a). This allows one to measure (or calculate) $u'^+(y^+)$ and u'_{OL} at a moderate Re and determine the statistical properties of $u^*_{\text{MHM}}(t^+, x^+, y^+, z^+)$. Since these properties are Re-independent, one can then measure only u'_{OL} in flight and determine the in-flight statistical properties of $u'^+(y^+)$ from (2). Note that (1) and (2) are incompatible, since the statistical properties of u'_{OL} depend on Re. Chernyshenko *et al.* [3] proposed to replace (1) with the formula

$$u = u_{\tau_L}(t, x, z)\tilde{u}\left(\frac{tu^2_{\tau_L}}{\nu}, \frac{xu_{\tau_L}}{\nu}, \frac{yu_{\tau_L}}{\nu}, \frac{zu_{\tau_L}}{\nu}\right). \tag{3}$$

This amounts to replacing the friction velocity $\bar{u}_\tau$ with the large-scale-filtered friction velocity $u_{\tau_L}(t, x, z)$ in the definition of wall units. With an additional assumption that the fluctuations of $u_{\tau_L}(t, x, z)$ are small as compared to $\bar{u}_\tau$ allowing linearisation, (3) leads, among many others, to the following formulae

$$\alpha_u = \frac{\overline{u'_L(y^+)u'_L(y_o^+)}}{\overline{u'^2_L(y_o^+)}} \approx \frac{U(y^+) + y^+\frac{dU}{dy^+}(y^+)}{U(y_o^+) + y_o^+\frac{dU}{dy^+}(y_o^+)},$$



(a) (b)

Figure 2. (a) Pareto front of large-large and small-small correlation coefficients, where each point represent a combination of cut-off frequency and wave-numbers; (b) Comparison for r as given by the theory (line) and numerical simulation (points).

$$\gamma(y) = \frac{\overline{u'^{+}_{\tau_L}\left(u'^2 - \overline{u'^2}\right)_L}}{\overline{u'^{+\,2}_{\tau_L}}} = 2\tilde{u}^2_{rms} + y\frac{d\tilde{u}^2_{rms}}{dy} + \frac{\overline{u'^{+\,3}_{\tau_L}}}{\overline{u'^{+\,2}_{\tau_L}}}\left[\left(U^* + y\frac{dU^*}{dy}\right)^2 + \frac{1}{2}\frac{d^2\left(y^2\tilde{u}^2_{rms}\right)}{dy^2}\right] + \ldots$$

where U and u_{rms} are the mean velocity and the mean velocity fluctuation respectively. Function $\alpha_u(y^+)$ characterises the correlation between large-scale motion away from the wall and near the wall, while $\gamma(y^+)$ characterises the correlation between large-scale motion and the intensity of small-scale fluctuations. The very good agreement for α_u, see Fig. 1(b), demonstrates the potential of the theory. However, this good agreement is partially due to the structure of the formula, which ensures that both curves start at 0 and end at 1. For γ the formula ensures only that both curves start at 0, and the agreement becomes less good away from the wall. The present work found that the observed discrepancy of γ is mainly due to the linearisation of the quasi-steady hypothesis (3).

We will now clarify the main hypothesis of QSQH theory, separating it into two hypotheses, the first hypothesis expressed by (3), and the second hypothesis being that $\tilde{u}$ is independent of Re. Here we will deal with the first hypothesis only. The large-scale filter L acts on any function to give its large-scale component: $u_L(t,x,y,z) = Lu(t,x,y,z)$ is the large-scale component of $u(t,x,y,z)$. We assume that $LL = L$, that is that applying the filter twice is the same as applying it once, and that $Lf(y) = f(y)$, that is that there is no filtering in the wall-normal direction y. We clarify (3) by requiring that $Lu_{\tau_L} = u_{\tau_L}$ and that $\tilde{u}$ is the sum of a mean flow and a small scale function, which means that taking the large-scale filter of any function of $\tilde{u}$ and large-scale-filtered quantities is equivalent to taking the average of that function over any of the homogeneous directions or time with the large-scale quantities kept constant. We then verify these assumptions by the data from the direct numerical simulations of a channel flow at Re_τ=1000 [2]. We used a frequency and wave-number cut-off filter in Fourier-transform space corresponding to the time period $T^+_c = 5500$, the longitudinal length $X^+_c = 1000\pi$, and the spanwise length $Z^+_c = 125\pi$ (normalised with the channel half-width). These cut-off parameters were selected by multi-objective optimisation, with one objective being the correlation coefficient between the large-scale velocity at the outer probe placed at $y^+ = 100$ and the large scale velocity at the probe in close vicinity to the wall, and another objective being the correlation coefficient between the small-scale velocities at the same probes. We argue that for the assumptions of the QSQH theory to be more accurate the large-scale to large-scale correlation should be large while the small-scale to small-scale correlation should be small. Fig. 2(a) shows the Pareto front and the point we selected.

We derived the higher-order terms of the expansions of various quantities in terms of the amplitude of fluctuations of $u_{\tau_L}(t,x,z)$ and found that taking them into account will affect the results for γ but not α. We also should point out that the effect of linearisation might be stronger for other velocity components, as suggested by the results of [1]. Finally, one of the simplest relationships the linearised theory gives is that the large scales are fully correlated throughout the near-wall region: $r = \overline{u'_L(y^+)u'_L(y^+_o)}/\sqrt{\overline{u'^2_L(y^+)u'^2_L(y^+_o)}} \approx 1$. Fig. 2(b) shows the degree of deviation from this result. Since the linearisation has been found to be not responsible for this deviation, it can be concluded that it is due to the approximate nature of the QSQH theory, thus giving an estimate of its accuracy for the filter we used. In the talk we will report more comparisons and discuss physical implications of our results.

References

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