ON THE INSTABILITY OF FLOW IN A GROOVED CHANNEL

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Abstract: It is shown that flow in a channel with longitudinal grooves is subject to two types of instabilities. The modified classical TS instability represents the first class while an inviscid instability associated with the groove-induced flow modulations represents the second class. The second instability dominates for grooves with the appropriate wave numbers and amplitudes.

INTRODUCTION

It has been known since the original work of Reynolds [1] that surface roughness plays an important role in the laminar–turbulent transition. This problem has most frequently been studied in the context of the identification of conditions when the presence of roughness can be ignored, i.e. when the wall can be viewed as hydraulically smooth. The question of roughness effects in turbulent flows has been studied in the context of drag determination starting with [2,3]; see [4,5] for reviews. The term "roughness" is not well defined; the term "rough wall" only means that the wall is not smooth. One can use terms like "roughness", "wall corrugation", and "surface topography" interchangeably as they all have the same meaning. In order to arrive at meaningful conclusions one needs to remove this arbitrariness and begin with a precise description of the wall geometry. This goal looks like a mathematical contradiction as there are an uncountable number of possible roughness forms but, nevertheless, a general answer is sought. This apparent contradiction has been bypassed in experimental investigations by using artificially created roughness forms, e.g. sets of cones, spheres, prisms, parallelepipeds, etc., with different spatial distributions [6]. The most promising method for the mathematical description of the hydrodynamic properties of surfaces with arbitrary topographies relies on the reduced-order geometry model [7]. The geometric properties are categorized by projecting the surface geometry onto a convenient functional space, e.g. Fourier space, with the expectation that only a few leading Fourier modes representing the topography matter as far as hydrodynamics are concerned. This technique permits the identification of the features of the topography that have a decisive influence on the flow response, with irrelevant details removed from consideration. Indeed, it has been demonstrated that, in many instances, it is sufficient to use only the leading Fourier mode to capture the main physical processes with accuracy sufficient for most applications [7]. The work reported here uses the reduced geometry concept and is aimed at explaining the role which corrugations in the form of grooves parallel to the flow direction (Fig.1) play in the transition process.

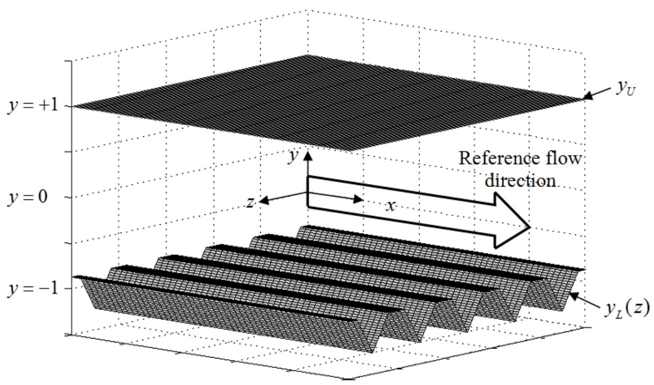

Figure 1. Sketch of the flow configuration.

PROBLEM FORMULATION AND DISCUSSION OF RESULTS

Consider flow in a channel with the smooth upper wall ($y_U = 1$) and grooved lower wall ($y_L = -1 + S\cos(\beta z)$). The stationary flow is described by the field equations of the form

$$\frac{\partial^2 u_B}{\partial y^2}+\frac{\partial^2 u_B}{\partial z^2}-Re\frac{\mathrm{d}p_B}{\mathrm{d}x}=0\,,\qquad u_B(y_L,z)=0\,,\qquad u_B(1,z)=0\,,\qquad Q=\lambda_z^{-1}\int_{z=0}^{z=\lambda_z}\int_{y=y_L(z)}^{y=1}u_B(y,z)\mathrm{d}y\,\mathrm{d}z=\frac{4}{3}$$

where u_B stands for the x-velocity component, p_B denotes pressure and Q stand for the flow rate. The linear disturbances of the form

$$\boldsymbol{V}_D(x,y,z,t)=\boldsymbol{G}_D(y,z)\,e^{i(\delta x+\mu z-\sigma t)}+c.c.\,,\qquad \boldsymbol{\omega}_D(x,y,z,t)=\boldsymbol{\Omega}_D(y,z)\,e^{i(\delta x+\mu z-\sigma t)}+c.c.$$

are added to the system, where $\boldsymbol{V}_D$ and $\boldsymbol{\omega}_D$ denote the disturbance velocity and vorticity vectors, δ and μ denote the real wave numbers in the x- and z-directions, respectively, $\sigma=\sigma_r+i\sigma_i$ is the complex amplification rate, σ_i is the rate of growth of disturbances, σ_r is the frequency of disturbances and $c.c.$ refers to complex conjugates. Amplitude functions $\boldsymbol{G}_D(y,z)$ and $\boldsymbol{\Omega}_D(y,z)$ have the form

$$\boldsymbol{G}_D(y,z)=\sum_{n=-\infty}^{n=+\infty}[g_u^{(n)}(y),g_v^{(n)}(y),g_w^{(n)}(y)]e^{in\beta z}+c.c.\,,\quad \boldsymbol{\Omega}_D(y,z)=\sum_{n=-\infty}^{n=+\infty}[g_\xi^{(n)}(y),g_\eta^{(n)}(y),g_\phi^{(n)}(y)]e^{in\beta z}+c.c.\,.$$

The dispersion relation for σ, Re, δ, μ is solved numerically.

Figure 2 displays a typical stability diagram. The TS waves have characteristics weakly affected by the grooves. The new instability mode has significantly lower critical Reynolds number which is a strong function of the groove amplitude. This mode is expected to dominate the transition process when grooves with the proper amplitude and wave number are present.

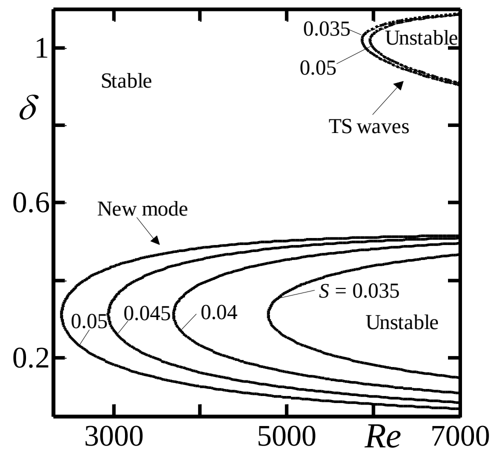

Figure 2. The neutral curves in the (Re, δ)-plane for the two-dimensional disturbances in a channel with β = 0.7. Solid and dotted lines correspond to the new mode and the TS waves, respectively.

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