LARGE-SCALE PATTERNS IN TURBULENT RAYLEIGH-BÉNARD CONVECTION IN VERY LARGE ASPECT RATIO CELLS

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Abstract Large-scale patterns, which are well-known from the spiral defect chaos regime of thermal convection at Rayleigh numbers $Ra < 10^4$, continue to exist in three-dimensional numerical simulations of turbulent Rayleigh-Bénard convection in extended cylindrical cells with an aspect ratio $\Gamma = 50$ and $Ra > 10^5$. They are uncovered when the turbulent fields are averaged in time and turbulent fluctuations are thus removed. We apply the Boussinesq closure to calculate turbulent viscosities and diffusivities, respectively. The resulting turbulent Rayleigh number Ra_*, that describes the convection of the mean patterns, is indeed in the spiral defect chaos range. Interestingly, the turbulent Prandtl numbers are smaller than one with $0.2 \leq Pr_* \leq 0.4$ for Prandtl numbers $0.7 \leq Pr \leq 10$. Finally, we demonstrate that these mean flow patterns are robust to an additional finite-amplitude side wall-forcing when the level of turbulent fluctuations in the flow is sufficiently high.

INTRODUCTION

The formation of regular patterns close to the onset of a hydrodynamic instability in spatially extended flows is well documented for generic cases. One of the most prominent examples are convection rolls in Rayleigh-Bénard flow heated from below and cooled from above [2]. Specifically linearly unstable systems with a sharp transition threshold to the convective flow state allow then for a perturbative expansion about the first unstable mode at onset. The expansion leads to an amplitude equation which is simpler than the original fluid equations and describes the formation of various patterns as a function of the system parameters. The inclusion of higher-order nonlinearities in the amplitude equation models higher-order bifurcations to increasingly complex flow patterns, such as spirals or defects [3]. This regime is known as spiral defect chaos (SDC). When the temperature difference across the fluid layer is further increased, the fluid motion becomes strongly time-dependent and eventually turbulent. The convection flow crosses over from the weakly nonlinear to the soft turbulence regime. The dimensionless Rayleigh number Ra describes the thermal driving. Turbulence is characterized by an irregular, stochastic and three-dimensional fluid motion. Does this however imply that the spiral patterns for velocity and temperature which are documented in the SDC regime at lower Rayleigh number disappear? If not, how can these patterns be extracted? How robust are they with respect to variations of the Prandtl number? And finally, how robust are they with respect to an additional side wall-forcing which is added to the momentum equation? These are the questions which we want to address in the present work by means of direct numerical simulations in extended cylindrical convection cells.

NUMERICAL MODEL

We perform DNS of the three-dimensional Boussinesq equations of thermal convection. An additional volume forcing is added to the momentum equation in order to test the robustness of the mean flow patterns. It is applied close to the side walls of the convection cell and designed such that it enforces the azimuthal symmetry in the vicinity of the side walls. The velocity field has a no-slip boundary condition on all walls. The temperature boundary condition is isothermal at the top and bottom plates and adiabatic at the side wall. The problem is formulated in cylindrical coordinates (r, ϕ, z) and solved by a second-order finite difference scheme. Following [1], we have chosen the DNS grid size Δ such that $\tilde{\Delta} = \max(\sqrt[3]{r\Delta_\phi \Delta_r(r)\Delta_z(z)})$ satisfies $\tilde{\Delta} \leq \pi\eta_K$, where η_K is the Kolmogorov dissipation length. The azimuthal grid size (Δ_ϕ) is equidistant, but the radial (Δ_r) and axial (Δ_r) grid sizes are non-equidistant. All runs start with the diffusive equilibrium state which is perturbed randomly.

RESULTS

Figure 1 shows a sequence of streamline plots viewed from the top for convection in air and $\Gamma = 50$. Panel (a) and a magnification in (b) are for $Ra = 5\,000$. For this Rayleigh number value almost no difference between an instantaneous snapshot and the time average was found which is taken over 100 free fall time units and not shown in the figure. The corresponding temperature pattern is displayed in panel (c). Panel (d) of the same figure and its magnification (e) display an instantaneous streamline plot at $Ra = 500\,000$. Both figures reflect the large amplitude of turbulent fluctuations. The fluctuating nature of the temperature field is also obvious in Fig. 1(f). The snapshots appear almost featureless. The bottom panels (g)–(i) show the time averages, which are obtained for a time interval of 200 free fall time units, and its magnification. The plots recapture now the typical spiral defect chaos (SDC) patterns as observed in panels (a)–(c) of the same figure for the Rayleigh number which is a hundred times smaller. A time average taken over τ has to be long enough

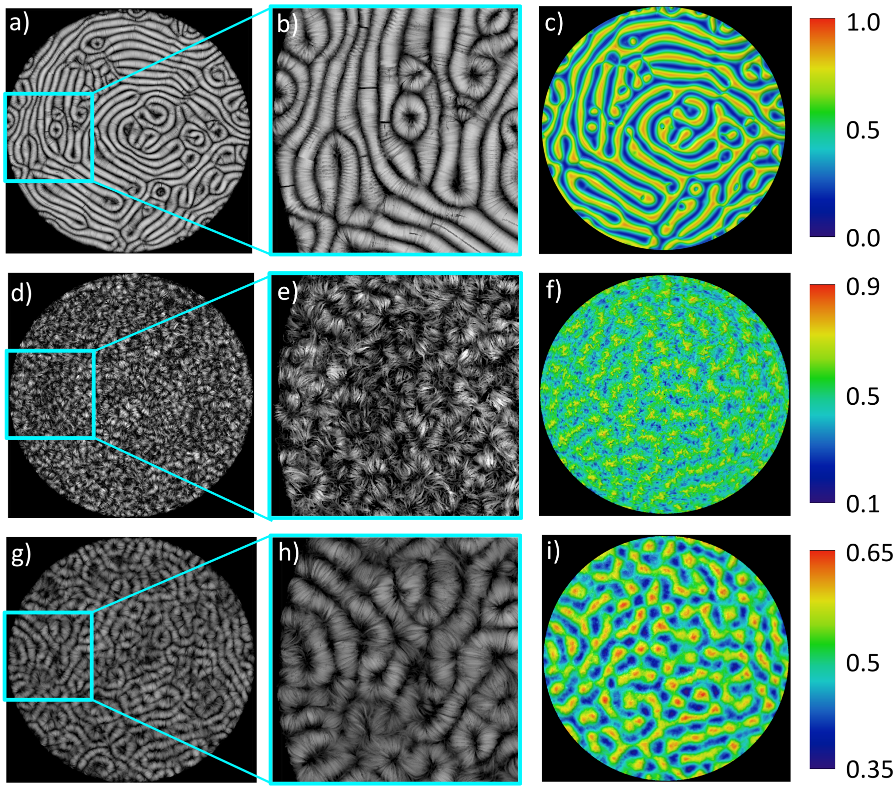

Figure 1. Streamlines of the velocity field (view from the top) and contours of the temperature field in a Rayleigh-Bénard convection cell. (a) Instantaneous velocity field pattern at $Ra = 5\,000$ and (b) magnification. (c) Corresponding temperature field in mid plane. (d,e) Instantaneous streamline plot and its magnification at $Ra = 500\,000$. (f) Corresponding temperature field in mid plane. (g,h) Streamline plot of the time-averaged velocity field at $Ra = 500\,000$ and magnification. (i) Corresponding temperature field in mid plane. The time average in (g)–(i) is taken over 200 free fall time units. All panels are for convection in air at a Prandtl number $Pr = 0.7$.

such that the turbulent fluctuations in the velocity field are suppressed. However, if the averaging procedure proceeds over a very long time interval then these patterns will be washed out for all Rayleigh numbers discussed here.

Our DNS demonstrate clearly that the well-known SDC patterns, which are known from the weakly nonlinear regime, continue to exist in the turbulent regime. They remain thus dynamically relevant and do not simply disappear when convection turns into the turbulent regime. The patterns are unraveled when the turbulent fluctuations are removed by time averaging, which is significantly smaller than the time scale over which the mean velocity and temperature patterns evolve. The latter time scale is of the order of 1000 free fall times. Our fully resolved simulations allow also to calculate the turbulent viscosities and diffusivities as well as related turbulent Rayleigh and Prandtl numbers, Ra_* and Pr_*. Their values fall indeed back into the range of the original SDC regime. The turbulent Prandtl numbers Pr_* vary between 0.2 and 0.4 and increase with increasing Pr. Our studies show also that the mean patterns are robust to finite-amplitude perturbations once the turbulent fluctuations in the flow are sufficiently large, i.e., once Pr at a given Ra is sufficiently small. This is demonstrated by a side wall forcing that sustained an azimuthally symmetric vortex.

References

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