# ACOUSTIC RADIATION DUE TO SCATTERING OF T-S WAVE BY THE MEAN-FLOW DISTORTION INDUCED BY STEADY LOCAL SUCTION

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*Abstract* Substantial sound waves can be generated by boundary-layer instability modes when the latter are scattered by a rapid mean-flow distortion. This is a rather generic mechanism and operates when an oncoming T-S wave is scattered by a steady local suction slot. This paper focuses on this problem by extending a recently developed Local Scattering Theory (Wu & Dong, J. Fluid Mech. submitted), where a so-called transmission coefficient, defined as the ratio of the T-S wave amplitude downstream of the scatter to that upstream, is introduced to characterize the effect of a local scatter on boundary-layer instability and transition. As in the earlier work, the mathematical formulation is based on triple-deck formulism, but in order to accommodate the acoustic far field, which was not considered in the paper mentioned, the unsteady terms in the upper deck, which play a leading-order role in radiation, are retained, and the influence of the radiated sound on the near-wall perturbation is included. The upper deck equation for the pressure is the Helmholtz equation rather than the Laplace equation. This leads to a modified pressure-displacement relation, which is coupled with the linearized boundary-layer equations in the lower deck. Discretization of the whole system formulates a generalized eigenvalue problem, which is solved numerically. It is found that suction suppresses oncoming T-S waves, and this effect increases with the suction velocity and the slot width. The directivity is independent of the flow parameters only when the Mach number is low. The intensity of the radiated sound in general increases with the frequency, the suction velocity and the width of the suction slot. Interestingly, for $O(1)$ suction velocities, the radiated sound is very weak, indicating that the gain of stabilizing effect does not cause aeroacoustic penalty.

## INTRODUCTION

Local suction at the wall surface is one of typical laminar flow control (LFC) strategies, and has attracted a great deal of interests at both theoretical and applied levels. If the flow is compressible, suction would not only alter the flow instability and transition, but also emit sound to the far field. The latter amounts to a problem in aeroacoustics. Wu & Dong[1] recently developed a mathematical framework to local scattering to account for the former effect. In this approach, a transmission coefficient is introduced to characterize the effect of a local scatter on instability. However, the compressible effect was neglected. This effect is considered in this paper so that the radiated sound can be predicted theoretically.

The mathematical description of the problem is based on triple-deck formulism, and most of the details can be found in Wu & Dong[1]. The key difference is in the upper deck, where unsteady terms, which play a leading-order role in radiation, is retained as suggested by Wu[2]. The governing equations are reduced to a Helmholtz equation for the pressure due to the inclusion of unsteadiness instead of the usual Laplace equation. Discretization of these equations formulates an eigenvalue problem, $\mathbf{A}\phi = \mathcal{T}\mathbf{B}\phi$, where $\mathbf{A}$ and $\mathbf{B}$ are coefficient matrices, $\phi$ is the vector consisting of unknown velocities and pressure, and $\mathcal{T}$ is the transmission coefficient. When the solution for the near-field hydrodynamics is obtained, the far-field sound can be calculated by the stationary-phase method.

## NUMERICAL RESULTS

The characteristic suction velocity is taken to be of $O(Re^{-3/8}U_\infty)$, and is expressed as $Re^{-3/8}U_\infty V_s(X)$, where $Re$ is the Reynolds number based on the free-stream velocity $U_\infty$ and the distance of the suction slot to the leading edge, and $X$ is the scaled variable in the triple deck theory. For illustration purpose, the suction velocity is taken to be Gaussian, $V_s(X) = V_{s0}\exp(-X^2/d^2)$, where $V_{s0}$ is the suction velocity at the slot center and $d$ characterizes the slot width.

Fig.1-(a) plots the displacement function $\tilde{A}$ (which measures the amplitude) *vs.* $X$. The disturbance evolves exponentially as $X \to \pm\infty$, but undergoes a sudden change in the vicinity of the suction slot. The effect of suction on the T-S waves can be characterized by the transmission coefficient $\mathcal{T}$. The effect is significant when the suction velocity is large. Fig.2-(b) shows the dependance of $\mathcal{T}$ on the frequency and suction velocity. Only slight differences are observed for the three frequencies of interest, implying that the $\mathcal{T}$ depends less on the frequency. However, it depends strongly on the suction velocity. For linear cases, in which $|V_{s0}| \ll 1$, $\mathcal{T}$ is about 1.0. As $|V_{s0}|$ increases, $\mathcal{T}$ decreases monotonously. For $|V_{s0}| = O(1)$, the $\mathcal{T}$ is less than 0.1, implying a significant stabilizing effect and hence delay of transition.

Since the acoustic pressure $p_s \propto r^{-1/2}$, where $r$ is the distance to the source, we introduce directivity, $Q = |p_s r^{1/2}|$, which describes the distribution of the sound with the direction. Fig.2-(a) displays the normalized directivity, $\tilde{Q} = Q/Q_{\theta=\pi}$, and the three curves for different frequencies agree quite well, implying that the directivity is largely independent of frequency. For sufficiently low Mach numbers, the directivity does not change with the suction velocity or slot width. For each case, the dominant radiation is in the upstream direction, and a silence angle about 77° is observed.

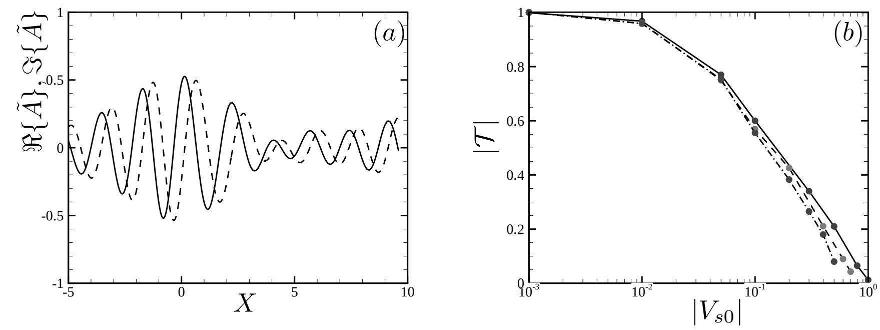


**Figure 1.** (a) The streamwise distribution of the real (solid line) and imaginary (dashed line) parts of the disturbance amplitude (displacement function) $\tilde{A}$ for frequency $\omega = 8.0$ and $V_{s0} = -1.0$; and (b) the dependance of the transmission coefficient $|\mathcal{T}|$ on $|V_{s0}|$ for $\omega = 4.0$ (solid line), 8.0 (dashed line) and 10.8 (dot-dashed line). For both plots, $(M, R, d) = (0.2, 10^6, 1.0)$.

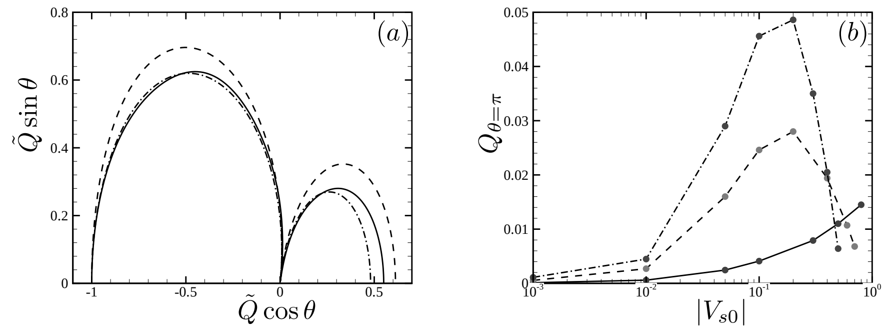


**Figure 2.** (a) The normalized directivity of the acoustic field as shown by plotting $Q$ in the polar coordinate for $V_{s0} = -0.001$; and (b) the intensity of the upstream propagating sound $Q_{\theta=\pi}$ *vs.* $|V_{s0}|$. The solid, dashed and dot-dashed lines represent the results for $\omega = 4.0, 8.0$ and $10.8$, respectively.

Fig.2-(b) shows the intensity of the sound propagating upstream i.e. in the direction of $\theta = \pi$. The intensity increases with the frequency, except for large suction velocities. It also increases with the suction velocity monotonically for relatively low (including near-neutral) frequencies, i.e. $\omega = 4.0$. However, for relatively large frequencies, the radiated sound intensity reaches the maximum at $V_{s0} = O(0.1)$, after which the intensity decreases. It follows that for $V_{s0} = O(1)$ the radiated sound is very weak, while at the same time the suction suppresses T-S waves considerably. This means that transition may be delayed without aeroacoustic penalty.

## CONCLUSION AND DISCUSSION

The acoustic radiation due to scattering of T-S waves by the mean-flow distortion caused by a local wall suction is investigated theoretically. A recently developed Local Scattering Theory[1] is extended by retaining unsteady effects in the upper-deck equations so that the radiation of sound to the far field can be described. Numerical solutions show that suction suppresses oncoming T-S waves, i.e. the transmission coefficient is less than unity. The stabilizing effect becomes more significant as the suction velocity and/or the slot width increases. For $M = 0.2$, the dominant acoustic wave radiated by the scattering is upstream propagating, and an angle of silence about $77^\circ$ is observed. The directivity does not depend on the frequency $\omega$, the suction velocity $V_{s0}$ or the slot width $d$, but the intensity does. The latter increases with $(\omega, V_{s0}, d)$ first, reaches the maximum and then decreases to a very level for $V_{s0} = O(1)$.

## References


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