BRAID ENTROPY OF FARADAY WAVES DRIVEN 2D TURBULENCE

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Abstract We report new experimental results that use tools from braid theory to characterize two-dimensional turbulent flows driven by Faraday waves. The average topological length $\langle L_{braid} \rangle$ of the material fluid lines is found to grow exponentially with time. It allows us to compute the braid's *topological entropy* S_{Braid}. We show that S_{Braid} increases as the square root of the turbulence kinetic energy $E \approx \tilde{u}^2$, where $\tilde{u}^2$ is the horizontal velocity variance . At long times, the PDFs of L_{braid} are positively skewed and present strong exponential tails.

TOPOLOGICAL BRAIDS IN FARADAY FLOWS

Recent studies have uncovered remarkable connections between the motion of particles on the surface of parametrically excited Faraday waves and the fluid motion in 2D turbulence [1, 2]. These similarities hinge on the ability of Faraday waves to generate horizontal lattices of vortices [3].

In these experiments, Faraday waves turbulence is formed in a vertically shaken container (circular, diameter=178mm), and in the deep water conditions. The forcing is monochromatic with a frequency set to $f_0 = 60$ Hz. Beyond a certain vertical acceleration threshold, parametrically forced Faraday waves appear with a dominant frequency $f = f_0/2$ and a wavelength $\lambda = 8.8$ mm. The forcing scale of the horizontal fluid motion is roughly $\lambda/2$. The liquid surface is seeded with particles to visualize the horizontal fluid motion. Particle tracking velocimetry technique is used to measure simultaneously the Lagrangian trajectories of hundreds of particles in the horizontal *(x-y)* plane [1, 4]. A few of these two-dimensional *(x-y)* Lagrangian trajectories are shown in Figure 1(a).

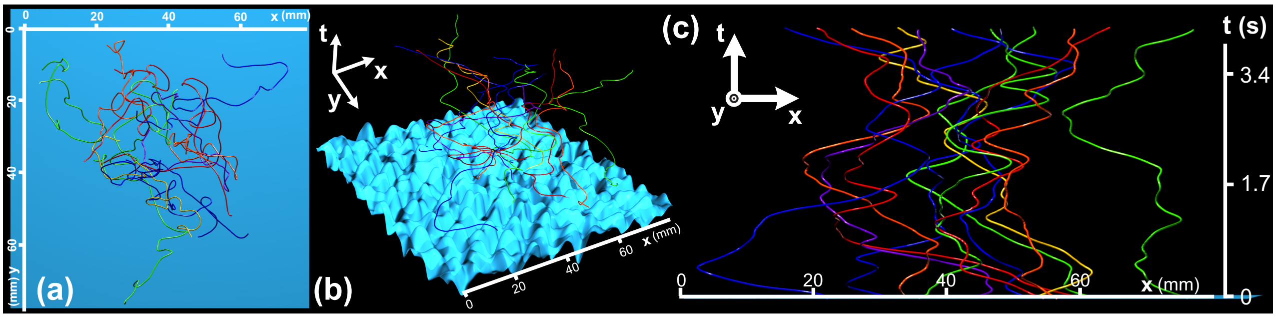

Figure 1. (a), Two-dimensional fluid particles trajectories tracked in the *(x-y)* plane for 4 seconds in a fully turbulent flow (horizontal kinetic energy $\tilde{u}^2 = 1e^{-3}\ m^2.s^{-2}$). (b), Perspective view of the three dimensional *(x-y-t)* strands built upon these trajectories. This figure also shows a 3D view of the topography of the disordered wavefield measured at $t = 0$ s. (c), The physical braid projected on the *(x-t)* plane. The braid is made of the 12 particle trajectories shown in (a).

Here, we use the topological braid description to characterize the degree of entanglement of particle trajectories in Faraday flows [5, 6, 7, 8]. This method is capable of capturing the deformation of fluid elements using a limited number of fluid particle trajectories. In this approach, two-dimensional trajectories are viewed as 3D strands, with time t being the third coordinate, Fig. 1(b). The braid picture relies on the projection of the 3D *(x-y-t)* trajectories onto the *(x-t)* (or *(y-t)*) plane, Fig. 1(c). In this plane, trajectories create a braid made of over- and under-crossings of strands. The braid entropy of the flow can then be deduced by considering the deformation of a topological fluid loop that is initially entangled in the braid [5]. This loop can neither intersect itself nor pass through the braid. In the course of time, each crossing along the braid distorts the loop and forces it to stretch or coil around the strands. Consequently, the sequence of crossings in a given braid allows computing a topological growth rate of the material lines in a flow. The length of this loop L_{braid} is also referred to as the braiding factor.

BRAID ENTROPY AND PDF OF TOPOLOGICAL LOOPS IN FARADAY TURBULENCE

In this analysis, the braids are made of 80 different particle trajectories. We measure the temporal evolution of the length L_{braid} of the fluid loop and take the statistical average $\langle L_{braid} \rangle$ over at least 10 different braids and 100,000 initial configurations for the topological fluid loop. We measure this topological feature for a broad range of the horizontal flow energy $u^2 = (10^{-5}, 2.10^{-3})\ m^2.s^{-2}$. The inset of Figure 2(b) shows an example of the kinetic energy spectrum of the horizontal velocity fluctuations in Faraday flows. The spectrum scaling is close to the Kolmogorov-Kraichnan prediction $E_k \propto k^{-5/3}$ at low wave number ($k < 1500\ m^{-1}$), it indicates the presence of the inverse energy cascade.

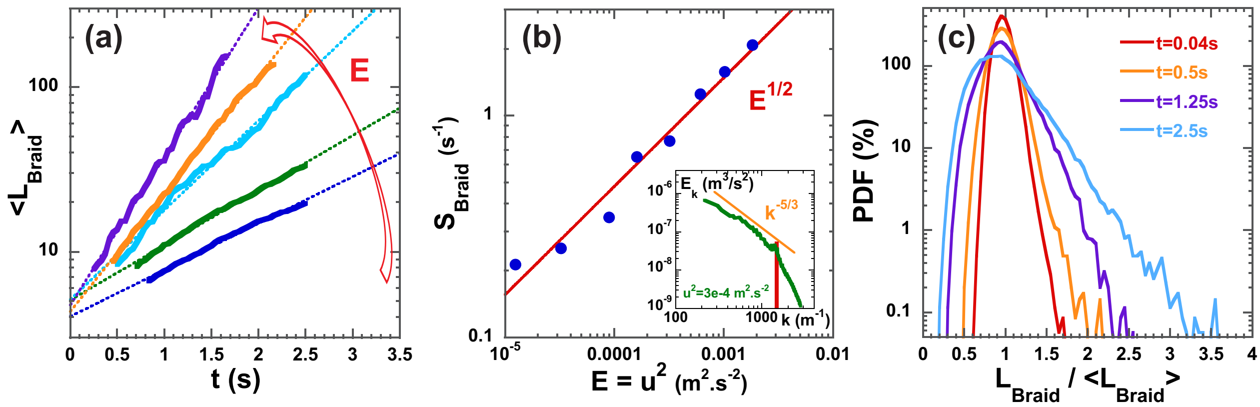

Figure 2. (a), Time evolution of the braiding factor $\langle L_{braid} \rangle$ versus time for various horizontal flow energy $E \approx \tilde{u}^2$. Dotted lines are exponential fits. (b), The braid entropy S_{Braid} versus the horizontal flow energy $E \approx \tilde{u}^2$. Inset: kinetic energy spectrum E_k of the horizontal velocity fluctuations measured at $\tilde{u}^2 = 3e^{-4}\ m^2.s^{-2}$. (c), PDFs of L_{braid} normalized by $\langle L_{braid} \rangle$ as a function of time for a fixed flow energy $\tilde{u}^2 = 1e^{-3}\ m^2.s^{-2}$. The PDFs are computed over 15 different braids and 100,000 different topological loops.

Figure 2(a) shows the temporal evolution of $\langle L_{braid} \rangle$ in Faraday flows as the flow energy is increased. After a transient state, $\langle L_{braid} \rangle$ grows exponentially with time and its growth rate increases with the flow energy. A braid entropy S_{Braid} can thus be measured as the growth rate of the logarithm of $\langle L_{braid} \rangle$. S_{Braid} increases as the square root of the flow energy $E \approx \tilde{u}^2$ as shown in Figure 2(b).

Figure 2(c) shows the probability distribution function (PDF) of the topological length L_{braid} of the fluid loop as a function of time. The braids used in this plot are obtained for a flow energy $\tilde{u}^2 = 1e^{-3}\ m^2.s^{-2}$. The PDFs are computed over 100,000 different initial configurations of the loop. The initial loop is randomly entangled in the braid, as a consequence, at $t = 0$ s the PDF is a gaussian function. After a transient period which depends on $\tilde{u}^2$ ($t > 0.5$ s for $\tilde{u}^2 = 1e^{-3}\ m^2.s^{-2}$), the PDFs become skewed and develop strong exponential tails at large values of L_{braid}.

In conclusion, S_{Braid} could be considered as a promising new measure of the level of irreversibility of 2D turbulence [9] based on topological considerations and sparse Lagrangian data. Much work has yet to be done to test the properties and potential applications of the braid entropy in fluid turbulence which is a paradigm for systems far from equilibrium.

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