# CONTACT VELOCITIES OF SMALL ELLIPSOIDS SETTLING IN TURBULENCE 

Christoph Siewert ${ }^{1}{ }^{\alpha}$, Rudie P. J. Kunnen ${ }^{2}$ \& Wolfgang Schröder ${ }^{1}$<br>${ }^{1}$ Institute of Aerodynamics, RWTH Aachen University, Aachen, Germany, c.siewert@aia.rwth-aachen.de<br>${ }^{2}$ Fluid Dynamics Laboratory, Eindhoven University of Technology, Eindhoven, The Netherlands<br>${ }^{\alpha}$ Corresponding author at: Laboratoire J.-L. Lagrange, CNRS, Observatoire de la Côte d'Azur, Nice, France


#### Abstract

Collisions of small and heavy non-spherical particles settling in turbulence are very important for systems such as ice clouds and proto-planetary disks where the particle spectra evolution is strongly dependent on the collision induced growth rate. Still, the influence of the particle shape on the collision probability is virtually unknown. Building on our recent investigation on the collision rate of monodisperse suspensions of ellipsoidal particles (Siewert et al., J. Fluid Mech. 758, 686-701, 2014), we show theoretically and by direct numerical simulations that the behavior of ellipsoids subject to turbulence and gravity is different from the behavior of spheres. Due to the dependence of the particle settling velocity on the particle orientation, the relative velocity at contact is influenced by turbulence. When ellipsoids differ either by mass or shape, their contact velocity is randomized by the randomized particle orientation. For particles much heavier than the fluid these orientation dependent settling velocity differences are larger than the relative velocities directly induced by the turbulent fluctuations.


## INTRODUCTION

During the last decade one field of very active research was the estimation of collision rates of small and heavy spherical particles suspended in a turbulent flow. Motivated by several problems involving collision induced particle growth such as droplet growth in warm clouds [8] and planetesimal formation in proto-planetary disks [14], a large number of numerical $[12,13,3]$ and theoretical $[6,5,2]$ studies has been conducted on inertial spheres in turbulence. Several interaction phenomena of particle inertia with fluid turbulence and gravity have been identified increasing the collision rate of small particles. Most of these findings are now experimentally verified [1, 9, 4]. Thus, it is accepted that turbulence can dramatically accelerate the particle spectra evolution since the growth rate of the very small particles in the beginning is moderately enhanced [15].
However, the collision rates of non-spherical particles in turbulence are virtually unknown. We recently showed that the collision rate of monodisperse suspensions of ellipsoids is drastically increased compared to spheres of the same mass due to an order of magnitude increased relative velocity at contact [11]. In this study, we show results for the bidisperse collision rate of ellipsoids of different size or shape.

## THEORY

Very often the density of particles suspended in a turbulent fluid flow is much higher than the fluid density. For liquid or frozen water particles and many types of solid particles in turbulent air a density ratio of the order of 1000 is typical. Hence, the gravity induced settling velocity $v_{t}$ takes on large values, i.e., is large compared to the turbulent fluctuation velocities even if the particles are smaller than the smallest turbulent scale, the Kolmogorov scale $\eta_{k}$. However, for nonspherical particles these settling velocities are orientation dependent. Fig. 1 shows the non-dimensional settling velocity and time scale of prolate ellipsoids at the aspect ratio $\beta=c / a=4$ as a function of the particle size. While the particle time scale remains small, the orientation dependent settling velocity is significantly higher than the Kolmogorov velocity scale at this moderate turbulence intensity. Even the orientation dependent spread is on the order of the Kolmogorov velocity scale, thus the orientation distribution is more important than the turbulent velocity fluctuations. The orientation distribution of inertial non-spherical particles is a very complex problem itself [10]. However, like for spheres it can be argued that the motion of ellipsoids of different size is basically uncorrelated such that the clustering or preferential concentration effect is only of minor importance [7]. Hence, the orientation at contact can be assumed to be random and the collision rate can be provided by calculating the relative velocities of randomly orientated particles settling in a quiescent flow. This is shown exemplarily in fig. 2.

## SIMULATION

To confirm our arguments, we conducted direct numerical simulations of a streamwise decaying turbulent flow and tracked several million one-way coupled particles by a Lagrangian point particle model based on Stokes flow conditions [7, 10, 11]. The relative velocity at contact is sampled assuming contact at separations of spheres at equivalent volume as the ellipsoids [11]. The results are also shown in fig. 2. It can be seen that the results match the theoretical predictions well. Hence, the direct contribution to the relative velocity by the turbulent fluctuations is negligible but they are important for the randomization of the particle orientations.


Figure 1. Dimensionless parameters characterizing ellipsoidal particles at an aspect ratio $\beta=c / a=4$ in a turbulent flow at a turbulent Froude number $F r_{t}=u_{k} / \sqrt{\eta_{k} g}=0.3$. Settling velocity non-dimensionalized by the Kolmogorov velocity scale $S v=v_{t} / u_{k}(-)$ and particle time scale non-dimensionalized by the Kolmogorov time scale $S t=\tau_{p} / \tau_{k}(--)$. The gray shaded area represents the orientation dependent spread.


Figure 2. Non-dimensionalized radial relative velocities at contact $\langle | w_{r}| \rangle / u_{k}$ of a reference ellipsoids at $a / \eta_{k}=0.03$ and $\beta=4$ as a function of size of the collision partners at $\beta=1$. For comparison $<\left|w_{r}\right|>/ u_{k}$ calculated for the ellispoids settling randomly orientated in a quiescent fluid.

## CONCLUSION

The results highlight the importance of combined effects of turbulence and gravity on small inertial particles and narrow the importance of particle clustering in turbulence to perfectly spherical particles. We believe that the presented arguments apply not only to ellipsoids but to all kinds of non-spherical particles as the prerequisite of orientation dependent settling velocities is generally fulfilled by non-spherical particles.

## References

[1] A. Aliseda, A. Cartellier, F. Hainaux, and J. C. Lasheras. Effect of preferential concentration on the settling velocity of heavy particles in homogeneous isotropic turbulence. J. Fluid Mech., 468:77-105, 102002.
[2] O. Ayala, B. Rosa, and L.-P. Wang. Effects of turbulence on the geometric collision rate of sedimenting droplets. Part 2. Theory and parameterization. New J. Phys., 10:075016, 2008.
[3] J. Bec, L. Biferale, M. Cencini, A. S. Lanotte, and F. Toschi. Intermittency in the velocity distribution of heavy particles in turbulence. J. Fluid Mech., 646:527-536, 2010.
[4] Gregory P Bewley, Ewe-Wei Saw, and Eberhard Bodenschatz. Observation of the sling effect. New J. Phys., 15(8):083051, 2013.
[5] J. Chun, D. L. Koch, S. L. Rani, A. Ahluwalia, and L. R. Collins. Clustering of aerosol particles in isotropic turbulence. J. Fluid Mech., 536:219-251, 2005.
[6] G. Falkovich, A. Fouxon, and M. G. Stepanov. Acceleration of rain initiation by cloud turbulence. Nature (London), 419:151-154, 2002.
[7] R. P. J. Kunnen, C. Siewert, M. Meinke, W. Schröder, and K.D. Beheng. Numerically determined geometric collision kernels in spatially evolving isotropic turbulence relevant for droplets in clouds. Atmos. Res., 127:8-21, 2013.
[8] H. Pruppacher and J. Klett. Microphysics of clouds and precipitation. Kluwer Academic Publishers, 1997.
[9] Ewe-Wei Saw, Raymond A Shaw, Juan P L C Salazar, and Lance R Collins. Spatial clustering of polydisperse inertial particles in turbulence: II. Comparing simulation with experiment. New J. Phys., 14(10):105031, 2012.
[10] C. Siewert, R. P. J. Kunnen, M. Meinke, and W. Schröder. Orientation statistics and settling velocity of ellipsoids in decaying turbulence. Atmos. Res., 142:45-56, 2014. The 16th International Conference on Clouds and Precipitation.
[11] C. Siewert, R. P. J. Kunnen, and W. Schröder. Collision rates of small ellipsoids settling in turbulence. J. Fluid Mech., 758:686-701, 2014.
[12] K. D. Squires and J. L. Eaton. Preferential concentration of particles by turbulence. Phys. Fluids, 3A:1169-1178, 1991.
[13] L.-P. Wang, O. Ayala, B. Rosa, and W. W. Grabowski. Turbulent collision efficiency of heavy particles relevant to cloud droplets. New J. Phys., 10:075013, 2008.
[14] Jonathan P. Williams and Lucas A. Cieza. Protoplanetary disks and their evolution. Annu. Rev. Astro. Astrophys., 49(1):67-117, 2011.
[15] Yan Xue, Lian-Ping Wang, and Wojciech W Grabowski. Growth of cloud droplets by turbulent collision-coalescence. J. Atmos. Sci., 65(2):331356, 2008.

