DIRECT NUMERICAL SIMULATIONS OF TURBULENT MIXING LAYERS BETWEEN TWO FLUIDS OF LARGE DENSITY DIFFERENCE

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Abstract In numerous practical applications, shear layers exist between fluids of strongly differing densities. At high Atwood numbers, the large variations in density introduce important effects that have recently been observed in other flows (e.g., Livescu and Ristorcelli, J. Fluid Mech., **605**:145–180, 2008). To investigate the inertial variable density effects on the instability growth and structure of mixing layers, we perform very large Direct Numerical Simulations of planar mixing layers between two miscible fluids, each with different density. The DNS domain size accommodates large extents of mode pairings, based on the most unstable modes obtained from linear stability analysis. The results display the overall statistical effects on the turbulence and mixing, as well as the structural differences that occur as Atwood number is varied. In particular, significant asymmetries are introduced by the differences in the densities of the mixing layer streams.

INTRODUCTION

Turbulent shear-driven mixing layers involving binary mixtures of fluids in which the participating fluids have significantly different densities are common in practical applications. The Atwood number $A = (\rho_2 - \rho_1)/(\rho_1 + \rho_2)$ characterizes the density difference between the two fluids with densities ρ_1 and ρ_2. The Boussinesq approximation implies $A \approx 0$. While the single fluid or, at most, the Boussinesq cases have received much attention, studies examining the $A > 0$ (non-Boussinesq) case are scarce. Here, we focus on shear layers with miscible fluids, which have applications ranging from atmospheric and oceanic flows, to combustion (e.g. the mixing of hydrogen in air results in $A \approx 0.87$), and to astrophysical situations, such as ejections in the solar corona [5], where $A \approx 0.38$.

Specifically, we consider miscible shear-driven mixing layers in which the density variation is due to compositional change. The effects of density variations in shear layers have been studied for many years. For example, the 1974 Brown & Roshko experimental study [1] involved mixing layers between various gases, including nitrogen and helium ($A = 0.75$). Even though the results highlighted the importance of compositional changes, later studies have focused mainly on the high speed case, where density fluctuations arise from acoustic or thermal effects (e.g. [6]) and results concerning the effects of compositional variations remain scarce. Nevertheless, important non-Boussinesq effects at high Atwood numbers have been discovered in buoyancy-driven turbulence simulations by Livescu & Ristorcelli [3, 4]. In order to focus on the inertial (non-Boussinesq) effects in the variable density mixing layer and separate them from acoustic and thermal effects, we consider the case of a mixture of different density fluids, each individually incompressible. The equations describing such a flow can be derived from the fully compressible Navier-Stokes equations with two species, by taking the infinite speed of sound limit [2].

$$\rho_{,t} + (\rho u_j)_{,j} = 0 \tag{1}$$
$$(\rho u_i)_{,t} + (\rho u_i u_j)_{,j} = -p_i + \mu\left(u_{i,j} + u_{j,i} - 2/3\delta_{ij} u_{k,k}\right)_{,j} \tag{2}$$
$$u_{j,j} = -\mathcal{D} \ln(\rho)_{,jj} \tag{3}$$

Buoyancy-driven turbulence [3, 4] was studied with (1–3), but this is their first application to non-buoyant mixing layers. We use a temporal approach to simulate the mixing layer with a streamwise-periodic domain. The initial mixing layer mean velocity profile and density variation profile are the same hyperbolic tangent. All velocity components are initially disturbed within a thin layer near the interface, but density is undisturbed. The velocity difference ΔU is generated with a left-traveling ($-x$) lower (negative-y) stream with lower density and right-traveling upper stream with higher density. Free-stream densities are chosen to obtain Atwood numbers of 0.001, 0.25, 0.50, 0.75, and 0.87. The kinematic viscosity is the same for each fluid, and the diffusivity between fluids results in a constant Schmidt number of 1. Equations (1–3) are solved using a direct numerical simulation approach (all flow scales are solved without any subgrid modeling or filtering) with 6th-order accurate compact differences in the inhomogeneous direction and spectral differentiation in the other directions. The very large domain allows structure mergers to occur naturally while the mixing layer grows toward self-similarity. In terms of initial $A = 0$ inviscid linear stability modes, the domain accommodates 64 least-stable streamwise wavelengths. The spanwise domain is one-fourth that length, resulting in grid sizes up to $6144 \times 2048 \times 1536$.

RESULTS

Growth rates of mixing layer integral thicknesses are important statistics to assess the overall effect of the density ratio and how the evolution is modified by the difference in densities between fluid streams. Momentum thickness is frequently used to characterize the width of mixing layers and scale the transverse coordinate when comparing statistics. Typically, momentum thickness is defined on a per-unit-volume basis: $\delta_m = \frac{1}{\rho_0(\Delta U)^2}\int_\infty^\infty \overline{\rho}\left[\frac{1}{2}\Delta U - \tilde{U}(y)\right]\left[\frac{1}{2}\Delta U + \tilde{U}(y)\right]\ dy$. Momentum thickness can also be defined on a per-unit-mass basis: $\delta_{m,pm} = \frac{1}{(\Delta U)^2}\int_\infty^\infty \left[\frac{1}{2}\Delta U - \tilde{U}(y)\right]\left[\frac{1}{2}\Delta U + \tilde{U}(y)\right]\ dy$. When density differences are large, the definition of momentum thickness becomes important. Relative to $A = 0.001$ (which agrees well with previous single-fluid simulations [7]), the δ_m growth rate decreases significantly with increasing Atwood number (density difference between streams) (figure 1a). This is consistent with the trend observed in Ref. [6], though our mixing layer between streams of different incompressible species is fundamentally different from Ref. [6], where the streams had the same ideal gas and the density variation was produced by thermodynamic differences.

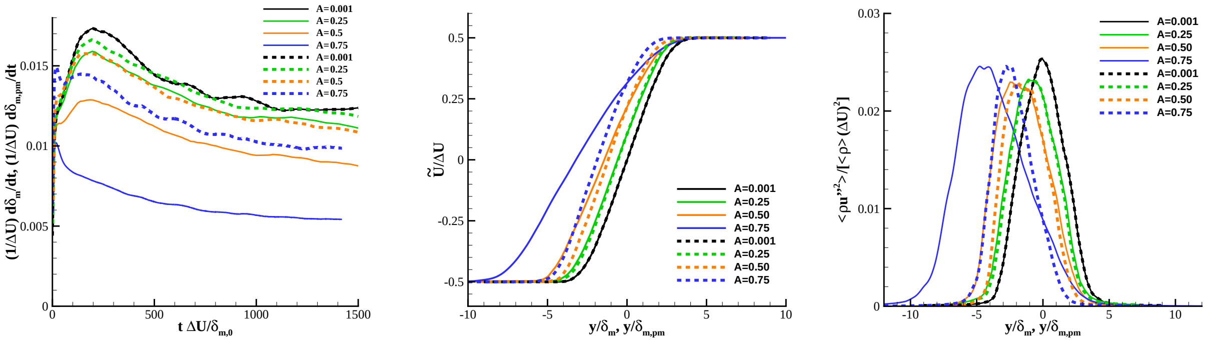

Figure 1. Left to right: (a) momentum thickness δ_m growth rate (solid lines) and $\delta_{m,pm}$ growth rate (dashed lines); late-time streamwise velocity (b) Favre mean and (c) turbulent stress intensity with y scaled by momentum thickness δ_m (solid) and $\delta_{m,pm}$ (dashed).

An important effect of density variation to the Favre-mean velocity profile is shown in figure 1b, in which the transverse coordinate is normalized by momentum thickness (solid lines) and momentum thickness per unit mass (dashed). There is a clear shift toward the lower density fluid stream, though the profiles are otherwise nearly identical between Atwood numbers using the latter ($\delta_{m,pm}$) normalization. Shifting is also apparent in the streamwise-component Favre-averaged turbulent stresses of figure 1c, while their peaks remain similar in magnitude for the range of Atwood numbers studied. Asymmetry at high Atwood number is most clearly seen in the structure of the mixing layers and can be observed by visualizing the density field (figure 2). At low A, density behaves as a passive scalar and the volume appears symmetric and consistent with single-fluid simulations. In the $A = 0.75$ case, the density variation results in more abundant fine-scale motions in lower-density bottom side and smoother structures in the higher-density top side.

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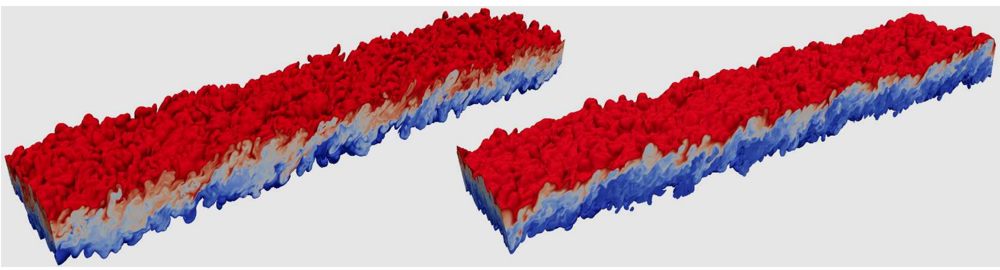

Figure 2. Contour visualization of the density field for (l) $A = 0.001$ and (r) 0.75.