TRANSITION AND TURBULENCE IN A WALL BOUNDED CHANNEL FLOW AT HIGH MACH NUMBER

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Abstract: The flow in a 3D wall bounded channel, simulated using the direct simulation Monte Carlo (DSMC) method, has been used as a test bed for examining different aspects of transition and turbulence at high Mach $M = U_m / (\sqrt{(\gamma k_B T_w/m)}$, and Reynolds numbers $Re = (\rho_m U_m H)/\mu_w$. Here, H is the channel half-width, U_m is the mean velocity, ρ_m is the mean density, T_w is the wall temperature, m is the molecular mass, μ_w is the molecular viscosity based on the temperature at the isothermal wall, and k_B is the Boltzmann constant.

The laminar-turbulent transition is accompanied by a discontinuous change in the friction factor even at high Mach number. The transition Reynolds number increases faster than linearly with Mach number, and the Knudsen number at transition (also proportional to the ratio of Mach and Reynolds numbers) passes through a maximum as the Mach number is increased. This maximum value is small, less than *0.009*, indicating that transition is a continuum phenomenon even at high Mach numbers.

In a high Mach turbulent flow, wall slip in the temperature and the velocities are significant. Slip occurs because the temperature/velocity of the molecules incident on the wall could be very different form that of the wall, even though the temperature/velocity of the reflected molecules is equal to that of the wall. There is slip even in the mean velocity as well as the intensity of the turbulent velocity fluctuations tangential to the wall.

In a compressible turbulent channel flow, we examine the result that the Kolmogorov scale, $\eta \sim (H\, Re^{-3/4})$ becomes asymptotically smaller than the mean free path, $\lambda \sim (H\ M/Re)$, for $M >> Re^{1/4}$. The simulation show that the ratio (mean free path to Kolmogorov scale) does decrease as $Re^{-1/4}$, but it does not increase linearly with Mach number. This is due to the decrease in the local Mach number within the channel, due to the increase in the temperature by viscous heating.

MOLECULAR SIMULATION

A molecular simulation technique, the direct simulation Monte Carlo (DSMC) algorithm [1], is used to study the different aspects of transition and turbulence in the flow in a 'channel' [2] [4], of width $2H$ bounded by plane walls at $y = \pm H$. The non-dimensional parameters are the Mach number, the Reynolds number, the Prandtl number, and the ratio of specific heats. Here the fluid is assumed to be an ideal gas with molecular diameter $d = 1.0 \times 10^{-10}$ m, molecular mass $m = 1.0 \times 10^{-26}$ kg/molecule, specific heat ratio $\gamma = 1.4$, Prandtl number $Pr = 0.7$. We use a variable hard sphere *(VHS)* molecular model with viscosity increasing as $T^{0.7}$, where T is the absolute temperature. Our study is restricted to a maximum Reynolds number of about 10^4 and a maximum Mach number of *30*. The DSMC simulations used here are fundamentally different from direct numerical simulations of the compressible Navier-Stokes equation. The DSMC method uses a probabilistic Monte-Carlo algorithm for simulating the Boltzmann equation for the velocity distribution function in dilute gases. Due to the use of the molecular chaos approximation in the Boltzmann equation, the simulation is restricted to dilute gases in the ideal gas limit. The DSMC method is preferred over the continuum Navier-Stokes equations both for compressible flows at high Mach numbers, where there could be significant errors in the solutions of the Navier-Stokes equations, as well as in the finite Knudsen number regime where the mean free path is comparable to the macroscopic length scale. In the DSMC algorithm, the evolution of the positions and velocities of 'simulated molecules', each of which represents a large number of real molecules, is followed in time and space. A probabilistic model, which reproduces correctly the variation of macroscopic properties, such as the pressure and viscosity, is used for the inter-molecular interactions. There are molecular effects which are inherently captured by the DSMC simulations which need to be modeled in simulations based on the Navier-Stokes equations. The DSMC simulations do predict a wall slip based on the molecule-wall collision laws, because even though the reflected molecules have the same mean velocity as the wall, the incident molecules do not necessarily have the same velocity. In a continuum simulation, it is necessary to model the wall slip. Previous DSMC simulations [3] have also shown the presence of temperature slip, that is, the gas temperature at the wall is different from the wall temperature. DSMC simulations can also be used for low Knudsen number flows, where the mean free path is comparable to the system size. Further, it is possible to compare the mean free path with other flow length scales, such as the Kolmogorov scale, because the mean free path can be explicitly calculated in the DSMC simulations.

TRANSITION AND TURBULENCE

Transition in a channel flow is usually inferred from a discontinuous change in the friction factor (scaled pressure drop) as the Reynolds number (or mean velocity) of the flow is increased. Here, the friction factor $\boldsymbol{f}$ is defined in terms of the

pressure gradient *(⊿p/L)* or the wall shear stress (τ_w) by the equation, $(\Delta P/L)\times(2H) = f\left((\rho_m U_m^2)/2\right)$, where *2H* is the channel width, ρ_m is the mean density, and U_m is the mean velocity. The plot of friction factor vs. Reynolds number for high Mach number flows is shown in *figure 1 (a), (b)*. It is clear that the discontinuous change in the friction factor is observed even at high Mach numbers, though both the transition Reynolds number and the value of the friction factor increase with Mach number. Also observed is the hysteretic behavior of the transition; the transition Reynolds number for the laminar-turbulent transition is higher than that for the turbulent-laminar transition as shown in *figure 1 (c), (d).*

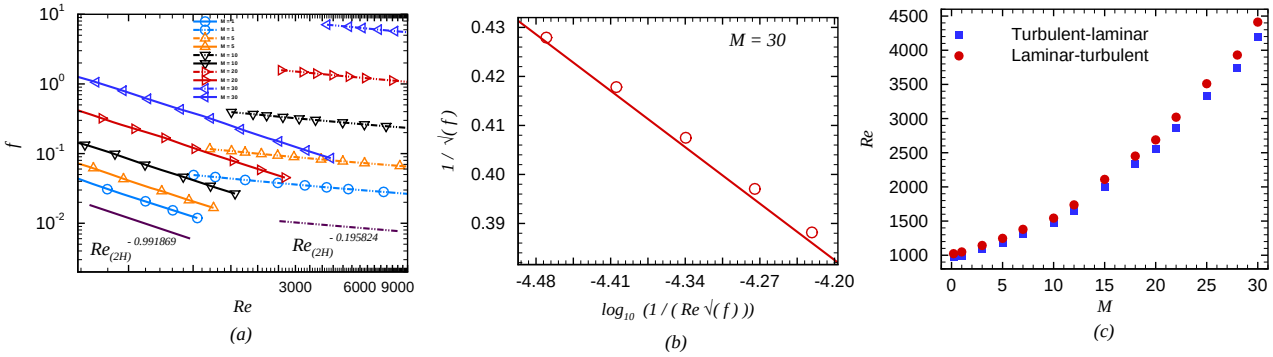
The velocity and temperature slip at the wall for mean and fluctuating components can be expressed as, $\langle u\rangle^s = l^s_{\langle u\rangle}\left(\langle\tau_{xy}/\mu\rangle\right)$, $u'^s = l^s_{u'}\left(\tau_{xy}/\mu\right)'$, $w'^s = l^s_{w'}\left(\tau_{xz}/\mu\right)'$, $\langle T\rangle^s = l^s_{\langle T\rangle}\left(\langle q/K\rangle\right)$, and $T'^s = l^s_{T'}\left(q/K\right)'$, where , $l^s_{\langle u\rangle}$, $l^s_{u'}$, $l^s_{w'}$ are the slip length for mean and fluctuating velocity component of *u*, and fluctuating velocity component of *w* respectively. $l^s_{\langle T\rangle}$, and $l^s_{T'}$ are the slip length for mean and fluctuating temperatures respectively. It is observed that with increasing Mach number slip length increases, as shown in *figure 1 (e), (f)*. The velocity fluctuations *(u'/U$_m$)*, *(v'/U$_m$)*, *(w'/U$_m$)*; density fluctuations *(ρ'/ρ$_m$);* and velocity vector for fluctuating velocities at $Re = 10^4$, $M = 30$ (turbulent state) on *(x − z)-plane at y/H = 0.95* are shown in *figure 1 (g), (h), (i), (j), (k).*

The variation of Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$ scaled by the channel width *H*, and the ratio of mean free path *(λ)* and the Kolmogorov scale *(η)*, are studied for different Mach numbers. There is a large variation in the Kolmogorov scale, by a factor of *2−3*, across the channel. However, the mean free path is nearly a constant across most of the channel, because the temperature is nearly constant. Due to this, *(λ/η)* varies by a factor of *2-3* across the channel. It is observed that the value of *(λ/η)* varies by a factor of *3* when the Mach number is increased from *3 to 30* as shown in *figure 1 (l).* However, when the local value of *(λ/η)* is scaled by its maximum value in the channel, we find a data collapse onto a single graph.

The classical turbulence scaling predicts that the ratio of the mean free path and the Kolmogorov scale, *(λ/η)*, is proportional to $(M/Re^{1/4})$. *Figure 1 (m)* shows that *(λ/η)*, averaged across the interior of the channel excluding *20%* of the volume adjacent to the walls, is indeed proportional to $Re^{-1/4}$ for the entire range of Mach numbers studied here. However, *figure 1(m)* shows that the ratio does not increase linearly with Mach number. This is due to the decrease in the local Mach number within the channel, due to the increase in the temperature by viscous heating.

A counter-intuitive result obtained is that the Kolmogorov scale could be smaller than the mean free path. However, the smallest length scale for gradients in the continuum approximation is not the mean free path, but the distance between molecules. The distance between molecules is proportional to $n^{-1/3}$, where *n* is the number density of the molecules. The mean free path, which is the average distance between successive collisions, is much larger than the distance between molecules, due to the small probability that two nearest molecules are moving towards each other. The ratio of the distance between molecules and the mean free path is proportional to $(n^{-1/3}/(nd^2)) \sim (nd^3)^{-4/3}$, which is proportional to $\varphi^{-4/3}$, where φ is the volume fraction of the molecules. Since the volume fraction is small for a dilute gas, the distance between molecules could be much smaller than the mean free path. We have verified, in the simulations, that the Kolmogorov scale is much larger than the distance between molecules. In *figure 1 (n)*, the ratio mean molecular spacing to Kolmogorov scale $1/(\eta\, n^{1/3})$ is plotted as a function of the wall-normal distance. It is clearly observed that this ratio is much less that *1*, indicating that the smallest length scale for the gradients is much smaller than the Kolmogorov scale.

RESULTS



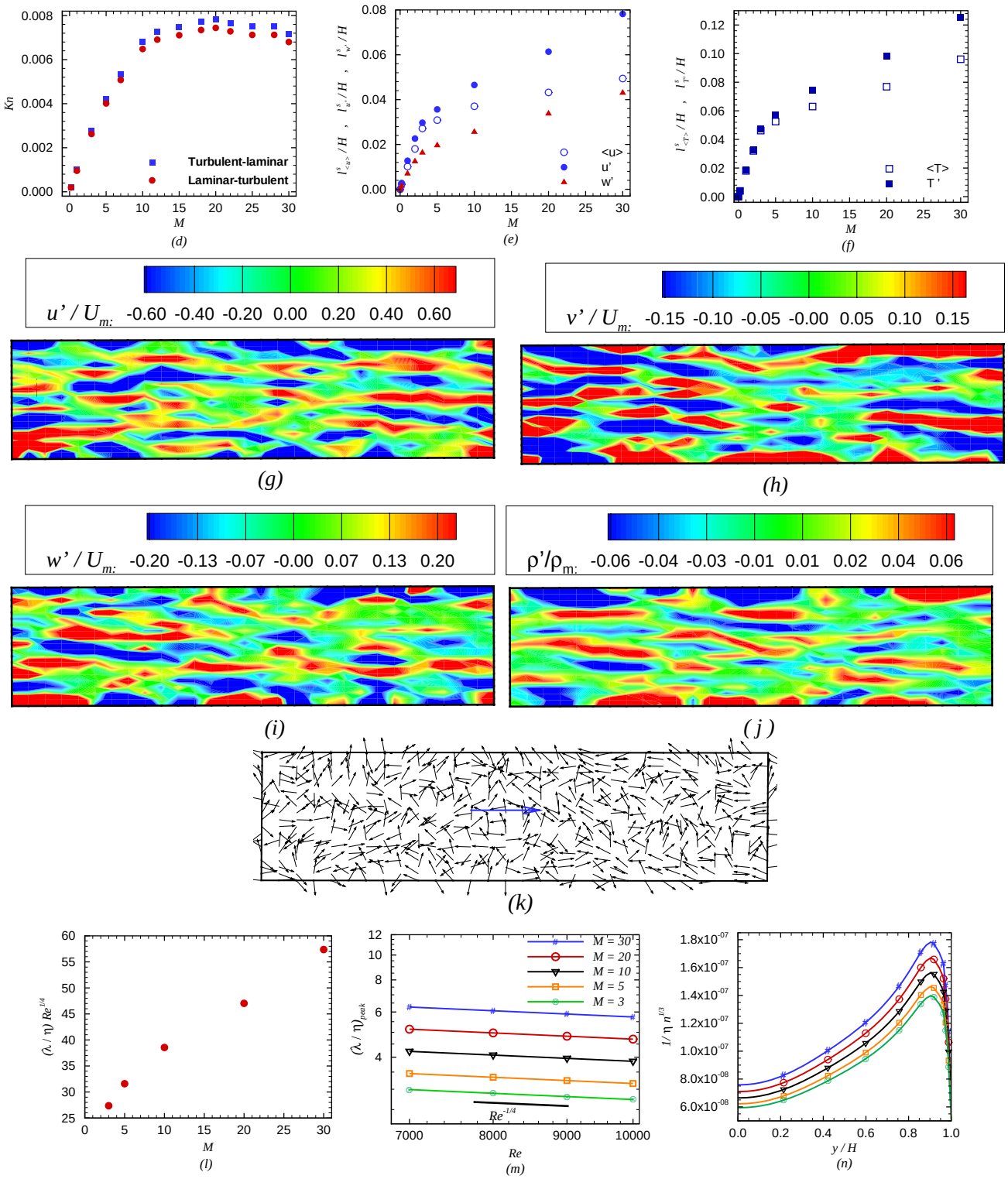


Figure 1. DSMC simulation results

References :

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