TURBULENT WAKES OF PLATES WITH NON-EQUILIBRIUM SIMILARITY SCALINGS

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Abstract We have conducted hot wire anemometry measurements of six different axisymmetric turbulent wakes which demonstrate the existence in all these wakes of non-equilibrium mean flow profile scalings and of the underlying self-preservation and non-equilibrium dissipation scalings. These mean flow profile scalings are different from those of all documented canonical boundary-free turbulent shear flows to date, all of which have been established for very far downstream regions.

The most basic and therefore arguably the most important property that any theory or model of turbulence must be able to predict is the mean flow profile. A model or theory of turbulence which can do this for a wide range of turbulent flows on the basis of only a few fundamental and robust assumptions is still lacking. However, in the case of canonical boundary-free turbulent shear flows such as turbulent jets, wakes and mixing layers, one can predict how the characteristic mean velocity difference and the characteristic size of the mean flow profile evolve with streamwise distance on the basis of two cornerstone assumptions (Townsend 1976, 'The structure of turbulent shear flow', Cambridge University Press). These two assumptions may not be sufficient for a complete prediction of the mean flow profile, but they do lead to some of its most important features. The first of these assumptions is self-preservation of one-point turbulence statistics and the second is the scaling of the turbulence dissipation rate (see Townsend 1976 and George 1989, in 'Advances in turbulence', Hemisphere).

The high Reynolds number scaling of the turbulence dissipation rate that is typically used is the one which is consistent with the Richardson-Kolmogorov equilibrium cascade. The resulting streamwise developments (power laws of the streamwise distance) of the characteristic mean velocity difference and the characteristic size of the mean flow profile can be found in many textbooks for many canonical boundary-free turbulent shear flows (e.g. Tennekes & Lumley 1972, 'A first course in turbulence', MIT press; Pope 2000, 'Turbulent flows', Cambridge university press; Townsend 1976). Recently, however, a new high Reynolds number dissipation law has been found in near-field grid-generated decaying turbulence which holds for many different types of grids and which characterises non-equilibrium small-scale turbulence in an apparently universal way (see Vassilicos 2015, Ann. Rev. Fluid Mech. 47, 95-114). The region where it holds can actually be substantially long depending on the turbulence-generating grid. Nedić, Vassilicos & Ganapathisubramani 2013, Phys. Rev. Lett. 111: 144503, used this non-equilibrium dissipation law in conjunction with the usual self-preservation hypothesis to derive mean flow profile scalings for non-equilibrium axisymmetric turbulent wakes. Specifically, their non-equilibrium predictions for the streamwise evolution (along x) of the centreline mean velocity deficit u_0 and the wake width δ are

$$u_0(x) = AU_\infty \left((x-x_0)/L_b\right)^{-1} (\theta/L_b)^2, \tag{1}$$

$$\delta(x) = B\sqrt{L_b(x-x_0)}, \tag{2}$$

where A and B are dimensionless constants, U_∞ is the incoming freestream velocity, L_b is a length-scale characterising the wake-generating object, θ is the momentum thickness and x_0 is a virtual origin. For comparison, the equilibrium predictions for axisymmetric turbulent wakes (see Townsend 1976 and George 1989) are $u_0(x) = AU_\infty \left((x-x_0)/\theta\right)^{-2/3}$ and $\delta(x) = B\theta\left((x-x_0)/\theta\right)^{1/3}$.

Nedić, Vassilicos & Ganapathisubramani (2013) confirmed these two predictions experimentally in the range $5L_b < x \leq 50L_b$ for mean flow profiles of axisymmetric and self-preserving turbulent wakes of thin plates with irregular edges placed normal to the incoming freestream (in which case L_b is the square root of the plate's frontal area $\mathcal{A}$). They chose plates with irregular edges to increase the turbulence Reynolds numbers and thereby increase their chances to detect mean flow signatures of the non-equilibrium dissipation scalings.

The presumed universality of the non-equilibrium dissipation law suggests that the non-equilibrium wake laws (1) and (2) should hold in any axisymmetric and self-preserving wake. The aim of the present contribution is therefore to demonstrate the validity of (1) and (2) in a variety of such wakes and to also confirm, for the first time, the presence in these wakes of the non-equilibrium dissipation law $\epsilon_0 = C_\epsilon K_0^{3/2}/\delta$ where K_0 is the centreline turbulent kinetic energy, ϵ_0 is its dissipation rate and C_ϵ is a dimensionless coefficient which is not constant as in equilibrium turbulence but proportional to the ratio of two Reynolds numbers, Re_G/Re_L. $Re_G \equiv U_\infty L_b/\nu$ is a global Reynolds number determined by the inlet conditions and $Re_L \equiv \sqrt{K_0}\delta/\nu$ is a local Reynolds number dependent on x.

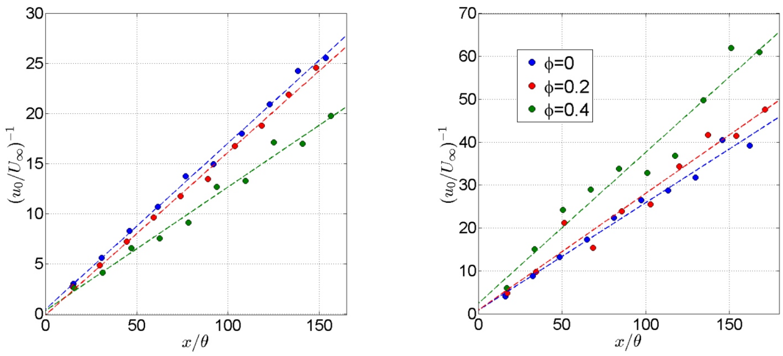
As the difference between the equilibrium and the non-equilibrium predictions involves the momentum thickness (e.g., equation (2) does not involve θ whereas the classical equilibrium scaling is $\delta(x) \sim \theta\left((x-x_0)/\theta\right)^{1/3}$), we have carried out wind tunnel anemometry measurements (single and cross wires) of wakes generated by plates that have different chord lengths, *i.e.* thicknesses in the direction of the flow. All the plates have therefore been manufactured with three different chord lengths l_c ($\phi = l_c/\sqrt{\mathcal{A}} \approx 0, 0.2, 0.4$) and were placed in the wind tunnel normal to the incoming flow. Three of our plates have simple square edge peripheries and the other three have irregular edge peripheries such as those of Nedić, Vassilicos & Ganapathisubramani (2013) who only considered $\phi \approx 0$. We have obtained the following four sets of definite results.

1. All six wakes are axisymmetric and self-preserving downstream of $x \approx 10L_b$.

2. The non-equilibrium wake scalings (1) and (2) hold irrespective of chord length and for both types of plates (i.e. for both regular and irregular edges) over a long way downstream (see figure 1).

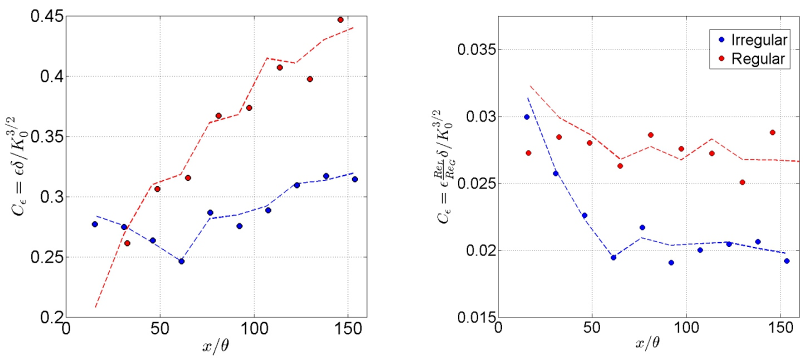
3. The centreline turbulence dissipation rate seems to follow the non-equilibrium law (figure 2b) $\epsilon = C_\epsilon^{Non-Eq}(Re_G/Re_L)K_0^{3/2}/\delta$ where C_ϵ^{Non-Eq} is a dimensionless constant at least for $x/\theta > 75$. The equilibrium law $\epsilon = C_\epsilon^{Eq} K_0^{3/2}/\delta$ with dimensionless constant C_ϵ^{Eq} is not supported by our data in the flow region considered.

4. The momentum thickness θ is about $0.30L_b$ for regular and $0.33L_b$ for irregular edges and does not significantly depend on normalised chord length ϕ. However, u_0 decreases/increases with ϕ for regular/irregular edges (figure 1), *i.e.* the dimensionless coefficient A in (1) decreases/increases with ϕ for regular/irregular plates. The inverse trends are observed for the wake width $\delta(x)$ in agreement with mean momentum conservation $U_\infty\theta^2 = u_0\delta^2$. The non-equilibrium mean flow profile has therefore some non-universal properties as it depends on chord and edge perimeter shape via the dimensionless coefficients A and B even though the non-equilibrium scalings (1) and (2) are universal.



(a) **(b)**

Figure 1. Inverse normalized velocity deficit $(u_0/U_\infty)^{-1}$ as a function of x/θ for the plates with irregular ***(a)*** and regular edges ***(b)***. In both figures the straight lines are the corresponding linear fits for the data as predicted by the scaling proposed in equation (1). The behaviour of the wake width is consistent with momentum conservation $U_\infty\theta^2 = u_0\delta^2$.



(a) **(b)**

Figure 2. Dissipation coefficients for both types of plates with $\phi = 0$. ***(a)*** Equilibrium dissipation coefficient $C_\epsilon^{Eq} = \epsilon_0\delta/K_0^{3/2}$. ***(b)*** Non-equilibrium dissipation coefficient $C_\epsilon^{Non-Eq} = \epsilon_0(Re_L/Re_G)\delta/K_0^{3/2}$. The dashed lines in both plots have been obtained by using fits of equation 2 for $\delta(x)$ instead of our direct measurements of $\delta(x)$.