SMALL-SCALE PROPERTIES OF TWO-DIMENSIONAL RAYLEIGH-TAYLOR TURBULENCE

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Abstract We report a high-resolution numerical study of small-scale properties of two-dimensional (2D) miscible Rayleigh-Taylor (RT) incompressible turbulence with the Boussinesq approximation at small Atwood number and unit Prandtl number. Our results show that the buoyancy force balances the inertial force at all scales below the integral length scale and thus validate the basic force-balance assumption of the Bolgiano-Obukhov scenario in 2D RT turbulence. We further examine other small-scale properties of 2D RT turbulence, such as temporal evolution of energy and thermal dissipation rates, the emergence of intermittency and anomalous scaling for high order moments of velocity and temperature differences, distributions of local dissipation scales, and so on.

INTRODUCTION

Turbulent mixing originated at the interface that separates two layers of fluids of different densities in a gravitational field, i.e. Rayleigh-Taylor (RT) instability, is ubiquitous in a variety of engineering, geophysical, and astrophysical systems. Although RT turbulence is of great importance and has been studied for many decades, there are still some open issues [1]. Specifically, for the past decade, many studies have focused on small-scale turbulent fluctuations in both two- (2D) and three-dimensional (3D) RT turbulence. In two dimensions, Chertkov [2] proposed a phenomenological theory. By assuming equipartition of the buoyancy and inertial forces at all scales in the inertial subrange for the energy equation, the model predicts a Bolgiano-Obukhov-like (BO59) [3] scaling for the cascades of both the velocity and temperature fields. This prediction was later confirmed from the view of structure functions by pioneering numerical simulations [4, 5]. In this paper, we want to deepen the previous studies by making a numerical simulation of RT turbulence in the 2D space, and our main focus is on the small-scale turbulent properties.

NUMERICAL METHOD

The time-dependent incompressible Oberbeck-Boussinesq equations of miscible RT turbulence in vorticity-stream function formulation, i.e.

$$\frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \nu\nabla^2\omega + \beta g\frac{\partial\theta}{\partial x}, \tag{1}$$

$$\nabla^2\psi = \omega, u = -\frac{\partial\psi}{\partial z},\ \ w = \frac{\partial\psi}{\partial x}, \tag{2}$$

$$\frac{\partial\theta}{\partial t} + (\mathbf{u}\cdot\nabla)\theta = \kappa\nabla^2\theta, \tag{3}$$

are solved in a 2D box of width L_x and height L_z with uniform grid spacing Δ_g. Here, $\theta(\mathbf{x},t)$ is the temperature field, proportional to the fluid density ρ via the thermal expansion coefficient β as $\rho = \rho_0[1-\beta(\theta-\theta_0)]$ (ρ_0 and θ_0 are reference values), $\mathbf{u}(\mathbf{x},t) = u\vec{x}+w\vec{z}$ the velocity field ($\vec{x}$ and $\vec{z}$ are the horizontal and vertical unit vectors, respectively), $\omega = \nabla\times\mathbf{u}$ the vorticity, ψ the stream function, g the gravitational acceleration, and κ the thermal diffusivity of the working fluid. Periodic boundary conditions for both velocity and temperature are applied to the horizontal direction, while for the top and bottom walls, no-penetration and no-slip velocity boundary conditions, and adiabatic (no flux) temperature boundary conditions are used. The direct numerical simulations are based on a compact fourth-order finite-difference scheme [6, 7]. The initial condition in our simulations is that the velocity is zero everywhere $\mathbf{u}(\mathbf{x},t=0)=0$, and the temperature varies as a step function of the vertical coordinate z, $\theta(\mathbf{x},t=0) = -sgn(z)\Theta_0/2$, with Θ_0 being the initial temperature jump that defines the Atwood number as $A = \beta\Theta_0/2$. RT instability is seeded by perturbing the initial condition with respect to the step profile. The interface $\theta = 0$ is perturbed by a superposition of cosine waves, $\cos(2\pi kx/L_x + \phi_k)$, with $30 \le k \le 60$, equal amplitude and random phases ϕ_k. The parameters of our simulations are listed in table 1. All statistical quantities studied in this paper are obtained by first calculating for each individual simulation and then averaging over all these realizations.

RESULTS

First of all, we study the growth of the mixing zone. We define here the width of the mixing zone, $h(t)$, as a z zone where $-0.4\Theta_0 \le \langle\theta(\mathbf{x},t)\rangle_x \le 0.4\Theta_0$ with $\langle\theta(\mathbf{x},t)\rangle_x$ being the mean vertical temperature profiles and $\langle\cdot\rangle_x$ a horizontal average. The measured mixing zone width $h(t)$ for run B is plotted as a function of t/τ in Fig. 1(a), where $\tau = \sqrt{L_z/Ag}$

Table 1. Parameters for the two types of RT runs. Ag, L_z, number of grid points N_x and N_z, Θ_0, viscosity ν, thermal diffusivity κ, Prandtl number $Pr = \nu/\kappa$, maximum Rayleigh number Ra_{max}, and number of individual realizations N_{conf}

	Ag	L_z	N_x	N_z	Θ_0	ν	κ	Pr	Ra_{max}	N_{conf}
run(A)	0.25	1	2048	8193	1	1.58×10^{-6}	1.58×10^{-6}	1	1×10^{11}	100
run(B)	0.25	1	4096	8193	1	2.89×10^{-6}	2.89×10^{-6}	1	3×10^{10}	32

is the characteristic time of the RT evolution. The dashed line in the figure marks the self-similar prediction of the t^2 growth law [1]. It is seen clearly that in the range $1.6 < t/\tau < 4$ two types of lines collapse roughly on top of each other, indicating a quadratic growth of $h(t)$. Further studies show that the power spectra (not shown here) of both the velocity and temperature files obtained within the self-similar range cover a broad range of scales [6], indicating that a turbulent state has been well developed. Therefore, we shall later analyze the temporal evolution of $Q(\eta)$ within this self-similarity time range.

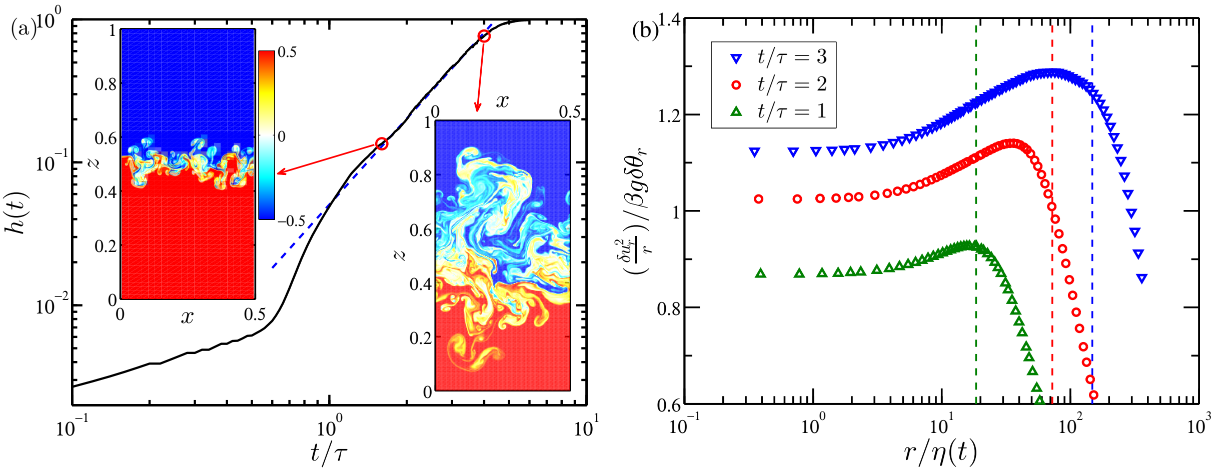

Figure 1. (a) The temporal evolution of the mixing layer width $h(t)$ for run B. The blue dashed line indicates the quadratic law $h(t) = \alpha Agt^2$ with $\alpha = 0.05$ obtained from the compensated plot of $h(t)/(Agt^2)$ (not shown here) for reference. Two snapshots of the temperature fields are shown at $t/\tau = 1.6$ (top left inset) and $t/\tau = 4$ (bottom right inset). Red and blue areas identify hot and cold regions, respectively. (b) Ratio of the inertial force $\langle \delta u_r^2/r \rangle_V$ to the buoyancy force $\langle \beta g \delta\theta_r \rangle_V$ as a function of the normalized scale $r/\eta(t)$ at times $t/\tau = 1$, 2, and 3 during the RT evolution for run A. The vertical dashed lines mark the corresponding integral length scales.

The basic assumption of the BO59 scenario is the force balance relation between the buoyancy force and the inertial force at all scales in the inertial subrange. In Fig. 1(b), we plot the ratio of the inertial force $\langle \delta u_r^2/r \rangle_V$ to the buoyancy force $\langle \beta g \delta\theta_r \rangle_V$ as a function of the normalized scale $r/\eta(t)$ for three distinct times in the self-similarity regime for run A. Here, $\delta\theta_r = |\theta(x+r,z) - \theta(x,z)|$ and $\delta u_r = |u(x+r,z) - u(x,z)|$ are temperature and horizontal velocity differences over a horizonal separation r, respectively. The vertical dashed lines in the figure mark the integral length scale of the horizontal velocity. One sees that the ratio is close to unity for all scales below the integral one for all three sets of data, implying an approximate equipartition between the buoyancy force and the inertial force and thus validating the force balance relation. Furthermore, it is seen that the ratio becomes larger at increasing time, indicating an increased magnitude of the inertial force with respect to the buoyancy force.

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