STABLY STRATIFIED SHEAR-PRODUCED TURBULENCE AND LARGE-SCALE WAVES IN A LID DRIVEN CAVITY

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Abstract We study experimentally stably stratified sheared turbulence and large-scale flows and waves in a lid driven cavity with a non-zero vertical mean temperature gradient. Geometrical properties of the large-scale vortex (e.g., its size and form) and the level of small-scale turbulence inside the vortex are controlled by the buoyancy (i.e., by the temperature stratification). The observed velocity fluctuations are produced by the shear of the large-scale vortex. At larger stratification obtained in our experiments, the strong turbulence region is located at the upper part of the cavity where the large-scale vortex exists. In this region the Brunt-Väisälä frequency is small and increases in the direction outside the large-scale vortex. This is the reason of that the large-scale internal gravity waves are observed in the regions outside the large-scale vortex. We found these waves by analyzing the non-instantaneous correlation functions of the temperature and velocity fields. The observed large-scale waves are nonlinear because the frequency of the waves determined from the temperature field measurements is two times smaller than that obtained from the velocity field measurements. The measured intensity of the waves is of the order of the level of the temperature turbulent fluctuations.

EXPERIMENTAL RESULTS AND COMPARISON WITH THE THEORETICAL PREDICTIONS

A number of studies of turbulent transport in lid-driven cavity flow have been conducted in the past, because the lid-driven cavity is encountered in many practical engineering and industrial applications, and serves as a benchmark problem for numerical simulations. Detailed discussions of the state of the art of the different studies of isothermal and temperature stratified lid-driven turbulent cavity flows have been published in several reviews (see, e.g. Refs. [1, 2]).

The main goal of this study is to investigate experimentally stably stratified sheared turbulent flows in a lid driven cavity in order to identify the parameters governing the mean and turbulent flows and to understand their effects on the momentum and heat transfer. The experiments have been carried out in a lid-driven turbulent cavity flow generated by a moving wall in rectangular cavity filled with air. Heated top wall and cooled bottom wall of the cavity impose a temperature gradient in the flow which causes temperature stratification of the air inside the cavity. The velocity field have been measured using a digital Particle Image Velocimetry (PIV) system. We have determined the mean and the r.m.s. velocities, two-point correlation functions and an integral scale of turbulence from the measured velocity fields. The temperature field has been measured with a temperature probe equipped with twelve E-thermocouples (for details see Ref. [3]).

These experiments demonstrate strong modification of the mean flow patterns with an increasing temperature difference ΔT between the hot top wall and the cold bottom wall of the cavity (that implies an increasing of the bulk Richardson number $\mathrm{Ri}_b = g\alpha\Delta T H_z/U_0^2$). Here α is the thermal expansion coefficient, $U_0 = 118$ cm/s is the lid velocity, $H_z = 24$ cm is the vertical height of the cavity and g is the gravitational acceleration. In isothermal flow the primary mean circulation (the main vortex) occupies the entire cavity, and two weak secondary mean vortexes are observed at the lower corners of the cavity. When ΔT is gradually increased, the position of the main vortex is shifted to the left, while the position of the right weak secondary vortex is shifted upwards and its size increases. Note that the top wall of the cavity moves in the left direction. At a further increase of the temperature difference the main vortex is pushed upwards, its size decreases, and the weak secondary mean flow is also strongly changed.

There are three regions in the mean flow for large stratification: a relatively strong mean flow in the upper part of the cavity including a main vortex in its left side, a mean sheared flow with a lesser mean velocity in its right side, and a very weak mean flow in the rest of the cavity. Therefore, when the strength of stable stratification increases, the main vortex tends to be confined to a small zone close to the sliding top lid and to the left wall of the cavity. The stable stratification suppresses the vertical mean motions, and, therefore, the impact of the sliding top wall penetrates to the smaller depth into the fluid. Temperature varies strongly in the bottom stagnant part of the flow with a stable stratification.

There are two regions in the flow where the magnitude of turbulent kinetic energy differs significantly for stronger stratifications. Strong (weak) turbulence is observed in regions with strong (weak) mean flow. For instance, in the case with the largest stratification $\Delta T = 54$ K $(\mathrm{Ri}_b = 0.29)$ obtained in our experiments, the turbulent kinetic energy in the strong turbulence region is by a factor 30 larger than that in the weak turbulence region. The strong turbulence is produced by the shear of large-scale vortex. The size of the shear-produced turbulence region due to the main vortex decreases with increase of the stratification.

To characterize the change in the mean flow pattern with the increase of the stratification, we determined the maximum vertical size L_z of the main (energy containing) large-scale circulation versus the temperature difference ΔT between the bottom and the top walls of the chamber obtained in the experiments. The length L_z is determined directly from the measured mean flow energy values as a location in the flow where the mean flow energy is by a factor 10 smaller than the maximum mean flow energy. We found that the maximum vertical size L_z of large-scale circulation is nearly constant when $\Delta T < 25$ K, and it decreases with ΔT as $L_z \propto 1/\Delta T$ for $\Delta T > 25$ K.

This scaling can be understood on the base of the budget equations for the mean velocity and temperature fields. Indeed, using the budget equation for the mean kinetic energy $E_U = \rho \boldsymbol{U}^2/2$, we obtain that the change of the mean kinetic energy δE_U is of the order of the work of the buoyancy force, $\rho \boldsymbol{U}^2/2 \sim \rho\beta(\delta T)L_z$, where ρ is the fluid density, $\beta = g/T_0$ is the buoyancy parameter, T is the mean temperature with the reference value T_0, and δT is the change of the mean temperature over the size of the large-scale circulation L_z. On the other hand, the budget equation for the squared mean temperature, T^2, shows that the change of the mean temperature δT over the size of the large-scale circulation L_z is of the order of the temperature difference ΔT between the bottom and the top walls of the chamber. This yields the following scaling: $L_z \sim \boldsymbol{U}^2/\beta\Delta T$.

This implies that the size of the shear-produced turbulence region due to the main vortex decreases with increase of the stratification. In this region, the component of the mean velocity along the moving wall of the cavity, U_y, is nearly constant (it does not change with ΔT). Substituting the vertical size of the turbulence region, L_z, into the mean shear $S = dU_y/dz \sim U_y/L_z$, we obtain $dU_y/dz \sim U_y\beta\Delta T/\boldsymbol{U}^2 \sim \beta\Delta T/U$, where we have taken into account that $U_y \sim U$. Therefore, the value of shear increases with ΔT, and, consequently, the shear production rate $\Pi = \nu_{_T} S^2$ increases with the increasing of the stratification, where $\nu_{_T} \sim \ell_z u_z$ is the turbulent viscosity and ℓ_z is the integral scale of turbulence in the vertical direction. Now let us estimate the turbulent kinetic energy, $\rho\boldsymbol{u}^2/2$, using the budget equation for this quantity: $\rho\boldsymbol{u}^2/2 \sim \Pi\ell_z/u_z \sim \ell_z^2 S^2 \sim \ell_z^2(\beta\Delta T/U)^2$, where $\boldsymbol{u}$ is the turbulent r.m.s. velocity. Since the mean velocity is nearly constant for $\Delta T > 25$ K, and $u \sim \ell_z\beta\Delta T/\sqrt{\rho}U$ (see the above estimate), the turbulent velocity increases with the increase of the stratification. Here we have taken into account the results of our measurements which show that the integral scale of turbulence in vertical direction does not change strongly with the change of ΔT when $\Delta T > 25$ K.

The internal gravity waves with the frequency $\omega = Nk_h/k$ can be excited in stably stratified flows, where k is the wave number, k_h is the horizontal wave number and $N = (\beta\,\nabla_z T)^{1/2}$ is the Brunt-Väisälä frequency. In our experiments in the region of the cavity with a weak turbulence we observed the large-scale internal gravity waves with the period of about 22 seconds. In particular, we found that the normalized one-point non-instantaneous correlation function $R(\tau) = \langle\delta T(z,t)\delta T(z,t+\tau)\rangle/\langle\delta T^2(z,t)\rangle$ of the large-scale temperature field determined for different z versus the time τ has a form of the Lorentz function, $R(\tau) = \exp(-\tau/\tau_0)\cos(\omega_0\tau)$ with $\tau_0 = 13$ s and $\omega_0 = 0.286$ s^{-1}, which corresponds to the period of the wave $2\pi/\omega_0 = 22$ seconds. Here $\delta T = \overline{T} - \overline{T}_0$, and $\overline{T}$ is the sliding averaged temperature (with 4.85 seconds window average), $\overline{T}_0 = \langle\overline{T}\rangle^{(sa)}$ and $\langle ... \rangle^{(sa)}$ is the 6.7 minutes average. Such form of the correlation function $R(\tau)$ indicates the presence of the large-scale waves with random phases. The memory or correlation time for these waves is about 11 s. In our analysis the temperature field is decomposed in three different parts: small-scale temperature fluctuations, the mean temperature field and the large-scale temperature field corresponding to the large-scale internal gravity waves.

We also performed similar analysis for the vertical large-scale velocity field. In our analysis the velocity field is decomposed in three different parts: small-scale velocity fluctuations, the mean velocity field and the large-scale velocity field corresponding to the large-scale internal gravity waves. Comparison of the normalized one-point non-instantaneous correlation function, $R_u(\tau)$, of the vertical large-scale velocity field with that of the temperature field, $R(\tau)$ shows that for short time scales ($\tau < 10$ s) these correlation functions are different. This implies that for these time-scales the wave spectra of the large-scale velocity and temperature fields are different. Indeed, the spectral analysis of these correlation functions shows a single peak for the vertical large-scale velocity and the large-scale temperature field, with 2:1 frequency ratio for these fields. The reason for such behaviour may be a parametric nonlinear excitation of these waves and interaction of the wave temperature and velocity fields. This also can be an interaction of nonlinear internal gravity waves and the large-scale Tollmien-Schlichting waves in sheared turbulent flows (see Ref. [4]). The observed features can be interpreted as a combination of standing and propagating waves which can be excited by the interaction of the mean flow and the walls of the cavity.

We found that the level of the intensities of turbulent temperature fluctuations are of the same order as the energy of the large-scale internal gravity waves. These turbulent fluctuations are larger in the lower part of the cavity where the mean temperature gradient is maximum. In the upper part of the cavity the shear caused by the large-scale circulation is maximum, and the mean temperature gradient is decreased.

References

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