TURBULENT ANNULAR PIPE FLOW IN SUBCRITICAL TRANSITION REGIME: EFFECT OF RADIUS RATIO ON STRUCTURES

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Abstract Direct numerical simulations are performed for annular Poiseuille flows with various radius ratios η (= $r_{\mathrm{in}}/r_{\mathrm{out}}$) ∈ [0.1, 0.8] in subcritical transition regime of $\mathrm{Re}_\tau \in [48, 150]$. Because the flow system of the annular Poiseuille flow can be regarded as the Hagen-Poiseuille flow (when $\eta \approx 0$) and the plane Poiseuille flow ($\eta \approx 1$), the (dis-)similarity and the connection between them are discussed. We found the occurrences of localized structure like the turbulent puff in the Hagen-Poiseuille flow at low η and of helical wave like the turbulent stripe pattern in the plane Poiseuille flow at high η. The switching point between helical wave and localized structure by changing η is caught in the region $\eta \in [0.1, 0.3]$.

INTRODUCTION

In recent years, the subcritical transition phenomena including the Hagen-Poiseuille flow (HPF) have been explained by much efforts of researchers. However, various types of flow systems such as the HPF and the plane Poiseuille flow (PPF) have been treated in individual, and a similarity and a connection among flow systems are still not discussed well. As for the issues regarding the subcritical transition, there are many theoretical researches to shed light on the nature of flow. In order to make another breakthrough, we here focus on a subcritical regime of the annular Poiseuille flow (APF) from the academic and practical interests. This flow has high demands for the engineering applications such as heat exchangers. The flow system of the APF is dependent on the radius ratio η (= $r_{\mathrm{in}}/r_{\mathrm{out}}$). With low radius ratio $\eta \approx 0$, the APF can be regarded as the HPF, while the APF with $\eta \approx 1$ can be regarded as the PPF. A study on the APF by varying η would work as a bridge between the APF and the PPF in terms of the understanding the subcritical scenario. In the subcritical transitions, it is known that the intermittent structures called 'turbulent puff' and 'turbulent stripe pattern' occurs in the HPF and the PPF, respectively, which are structures with very long streamwise extent [1, 2]. Those intermittent turbulent states are accompanied by localized and self-sustained turbulence and a typical property is the spatial coexistence of two stable states of laminar and turbulence in a flow. On the HPF, the transitional phenomenon of the turbulent puff has been elucidated well and the critical Reynolds number for global instability (Re_g) has been resolved by scrutinizing the lifetime of turbulent puff [3]. Because the PPF has one more spanwise degree of freedom than the HPF and the turbulent stripe pattern is accompanied by spanwise secondary flow, Re_g has not been resolved on the PPF and there are remaining mysteries that are reason why it is inclined with respect to the spanwise direction, whether it is robust or not, and its lifetime is finite or infinite. Therefore, the study on the subcritical transition regime of the APF is expected to puzzle out that of the PPF by connecting the HPF.

The present paper reports on a characteristic turbulent structure in various η of the APF and we would discuss the similarity and dissimilarity among APF, HPF and PPF.

NUMERICAL CONDITION

We performed direct numerical simulations of the APF in the range of Re_τ (= $u_\tau d/2\nu$) = 48–150 and η = 0.1–0.8 (u_τ, mean friction velocity on the inner and the outer wall; d, the gap between the inner and the outer wall). We set up long streamwise domains of $L_x = 51.2d$–$172.0d$ to capture the intermittent structures with long streamwise extents. As for the azimuthal direction, our domain covered the whole domain of $\theta = 2\pi$ and then the length of azimuthal L_θ (= $r\theta$) is provided by η: for instance, ($L_\theta|_{\mathrm{in}}$, $L_\theta|_{\mathrm{ou}}$) = ($6.28d$, $12.56d$) at $\eta = 0.5$.

RESULTS

As seen in Fig. 1, at a transitional Reynolds number ($\mathrm{Re}_\tau = 72$), weak-fluctuation regions (similar to the laminar state) occur intermittently but regularly so that localized turbulence takes the form of a banded structure in $\eta = 0.8$, as in the PPF. It should be noted that, in the APF not also in the PPF, the turbulent stripe pattern is in a helical shape around the inner cylinder, since the APF is a closed system in the azimuthal direction. As Re_τ decreases, the turbulent region is localized further and some inclined turbulent patches are organized (figure not shown).

As the same with $\eta = 0.8$, for $\eta = 0.5$ and 0.3 (Fig. 2), localized laminar states occur below $\mathrm{Re}_\tau = 80$ and also the helical wave is observed at $\mathrm{Re}_\tau = 64$. Although it is known that the turbulent stripe pattern in PPF is a large-scale structure with a spanwise spacing of about 12 times of the channel width. It is interesting to note that the turbulent stripe pattern has occurred even in the flow system that is not enough size in the spanwise distance to capture the expected pattern and the difference between $L_\theta|_{\mathrm{in}}$ and $L_\theta|_{\mathrm{ou}}$ is more than doubled. However, the turbulent stripe pattern becomes localized especially near the inner cylinder because the effect of flow redirection from the streamwise to the spanwise direction is lower than large η, as a result of decreasing L_θ due to small η.

As shown in Fig. 3, the flow with $\eta = 0.1$ maintains fully-developed turbulence even at very low Re_τ, compared to the case of larger η. Although we cannot regard the stripe pattern nor the spanwise secondary flow, the localized turbulent structure similar to the turbulent puff are observed at $\mathrm{Re}_\tau = 56$ and 52. This structure exhibits a long streamwise extent same as the turbulent puff but the interval between turbulent patches is not regular.

SUMMARY

The dominant turbulent structure should switch between turbulent stripe pattern and localized structure like turbulent puff by changing η. For $\eta \geq 0.3$, the turbulent stripe pattern accompanied by spanwise secondary flow arises and a transition region is between $\eta = 0.1$ and 0.3. The Reynolds number when the turbulent stripe pattern emerges differs depending on η because the Reynolds number based only on the friction velocity on the inner cylinder $\mathrm{Re}_\tau|_{\mathrm{in}}$ is much larger than averaged friction Reynolds number: $(\mathrm{Re}_\tau|_{\mathrm{in}}, \mathrm{Re}_\tau|_{\mathrm{ou}}) = (73, 61)$ at $\mathrm{Re}_\tau = 64$ of $\eta = 0.3$; and $(\mathrm{Re}_\tau|_{\mathrm{in}}, \mathrm{Re}_\tau|_{\mathrm{ou}}) = (73, 71)$ at $\mathrm{Re}_\tau = 72$ of $\eta = 0.8$. At $\eta = 0.1$, the spanwise secondary flow disappears and localized structure crops up from $\mathrm{Re}_\tau = 56$. The Reynolds number range of the transitional regime is very narrow, because the flow turns into laminar state at $\mathrm{Re}_\tau = 48$.

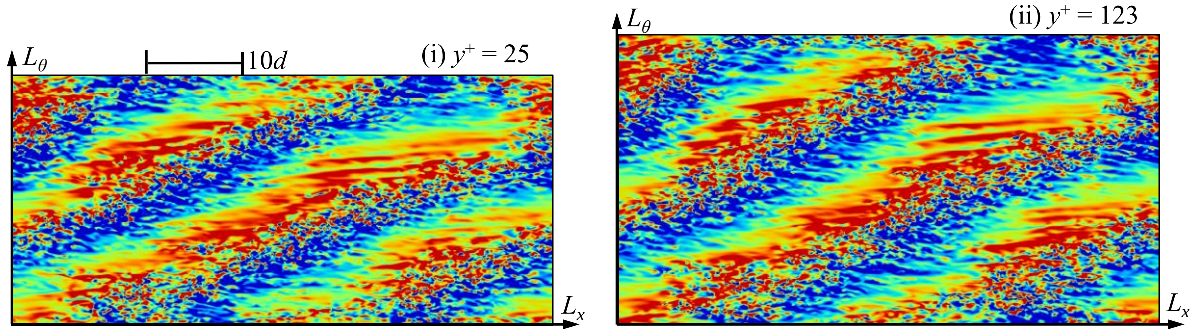

Figure 1. Contours of spanwise velocity fluctuation u_θ' in x-z planes at $y^+ = 25$ from (i) the outer cylinder and (ii) the inner cylinder for $\mathrm{Re}_\tau = 72$ on $\eta = 0.8$. Here, x represents the axial direction and z the azimuthal direction of the annular pipe. Red and blue regions indicate $u_\theta' = 1.0$ and -1.0, respectively. The mean flow direction is in x.

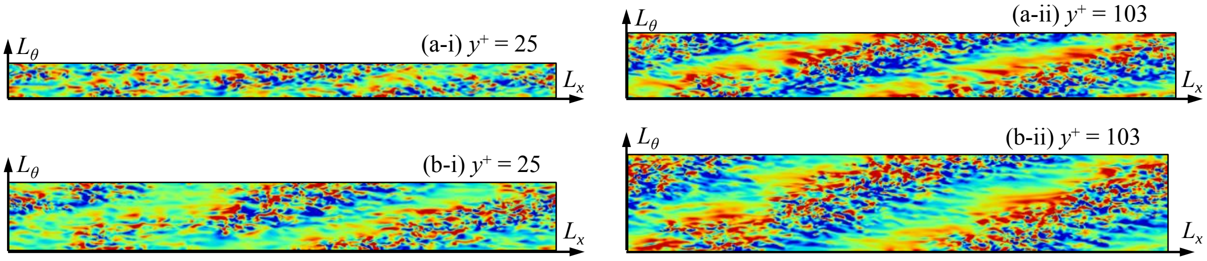

Figure 2. Same as Figure 1, but for $\mathrm{Re}_\tau = 56$ on η is (a) 0.3 and (b) 0.5.

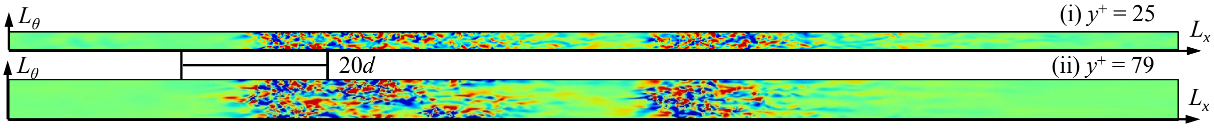

Figure 3. Same as Figure 1, but for $\mathrm{Re}_\tau = 52$ on $\eta = 0.1$.

References

[1] I. J. Wygnanski and F. H. Champagne. On transition in a pipe. Part 1. The origin of puffs and slugs and the flow in a turbulent slug. *Journal of Fluid Mech.* **59**: 281–335, 1973.

[2] T. Tsukahara, H. Seki, H. Kawamura and D. Tochio. DNS of turbulent channel flow at very low Reynolds numbers. *Proc. TSFP4,* Williamsburg, VA, 27-29 Jun.: 935–940, 2005; *arXiv*, 1406.0248.

[3] K. Avila, B. Moxey, A. de Lozar, M. Avila, D. Barkley and B. Hof. The onset of turbulence in pipe flow. *Science* **333**: 192–196, 2011.