MAGNETIC CONDENSATE IN 2D MHD

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Abstract By means of numerical simulations, we investigate the phenomenon of self-organization of the magnetic field in a large-scale coherent structure, which occurs in two-dimensional Magneto-Hydro-Dynamic (2D MHD) in presence of a magnetic forcing. We show that the magnetic condensate is not able to induce a large-scale motor effect on the velocity field.

INTRODUCTION

The equations for incompressible, two-dimensional MHD can be written in terms of the vorticity field $\omega = \boldsymbol{e}_z \cdot \nabla \times \boldsymbol{u}$, and the magnetic potential a, which is related to the magnetic induction field $\boldsymbol{b} = \nabla \times (\boldsymbol{e}_z a)$, as follows:

$$\partial_t \omega + \boldsymbol{u} \cdot \nabla \omega = \boldsymbol{b} \cdot \nabla j + \nu \Delta \omega + f_\omega \quad (1)$$

$$\partial_t a + \boldsymbol{u} \cdot \nabla a = \mu \Delta a + f_a. \quad (2)$$

where $j = \boldsymbol{e}_z \cdot \nabla \times \boldsymbol{b} = -\Delta a$ is the magnetic current, ν is the kinematic viscosity and μ is the magnetic diffusivity. Both the velocity and magnetic field are assumed to be free $\nabla \cdot \boldsymbol{u} = 0$, $\nabla \cdot \boldsymbol{b} = 0$.
In absence of forcing and dissipation ($f_\omega = f_a = 0$, $\nu = \mu = 0$), the system have two positive-defined quadratic invariants: the square magnetic potential $A = 1/2\langle a^2 \rangle$ and the total energy $E = E_u + E_b$, given by the sum of the kinetic energy $E_u = 1/2\langle |\boldsymbol{u}|^2 \rangle$ and the magnetic energy $E_b = 1/2\langle |\boldsymbol{b}|^2 \rangle$.
In the forced-dissipated case, when the system is sustained by a mechanical force acting on the vorticity field f_ω, and there is no forcing on the magnetic field $f_a = 0$, any initial fluctuation of the magnetic field decays because of the anti-dynamo theorem [1], and the system eventually recovers the usual 2D hydrodynamics, characterised by an inverse cascade of kinetic energy E_u and a direct cascade of enstrophy $Z = 1/2\langle \omega^2 \rangle$ [2].
The behaviour of the system is completely different in the opposite case in which there is only a magnetic force f_a and the mechanical force is absent $f_\omega = 0$. In this case one observes a dual cascade scenario, with a direct cascade of the total energy E and an inverse cascade of the square magnetic potential A [3]. When both the the kinetic and magnetic forcing are present, it is possible to observe a coexistence of inverse and direct cascade of energy [4].
Here we focus on the late stage of the evolution in the case in which there is only a magnetic force f_a, and the mechanical force is absent $f_\omega = 0$. In a bounded domain, the inverse cascade of A eventually gives rise to a condensed state, which should induce the self-organization of a large-scale intense magnetic field. An interesting question is whether or not the magnetic field of the condensed state is able to act as a "magnetic motor" and to induce a large-scale coherent flow.

RESULTS

We performed numerical simulations of Eqs.(1,2) in a periodic square domain of size $L_x = L_y = 2\pi$ at resolution $N_x = N_y = 512$. The system is forced by a magnetic force f_a, Gaussian and random in time, acting on a narrow wave-number shell $k_f \in [15 - 17]$, which provides a statistically steady input of magnetic potential ε_A and magnetic energy ε_E. For convenience, the viscous dissipative terms have been replaced by iper-viscous terms $-\nu_8 \Delta^8 \omega$, $-\mu_8 \Delta^8 a$.
In Figure 1 we show the temporal evolution of the magnetic potential A, which grows linearly in time as $A(t) \simeq \varepsilon_A t$. In the late stage of the simulation ($t > 20$) the magnetic potential field a organizes in a large-scale dipolar structure (see Figure 1). The temporal evolution of the magnetic and kinetic energy shows the formation of an almost steady regime at early times ($t < 20$) before the formation of the condensed state. In the late stage of the simulation ($t > 20$), the formation of the condensed state causes an increase of the magnetic energy, which grows linearly in time with the same rate of A, while the kinetic energy does not show a significant growth.
The development of the inverse cascade of magnetic potential A is clearly visible in the the early stage of the evolution of the spectrum of magnetic potential $A(k)$ (see Figure 2). A spectrum $A(k) \sim k^{-7/3}$ is observed for $k < k_F$. In the late stage the spectrum shows the accumulation of A in the mode $k = 1$, which correspond to the dipolar structure shown in Figure 1.
It is interesting to note that in the late stage, also the spectrum of magnetic energy $E_b(k)$ shows a clear accumulation in the largest mode $k = 1$ (see Figure 2). On the other hand, the kinetic energy spectrum $E_u(k)$ does not show an accumulation in the mode $k = 1$. Actually, in the late stage of the evolution the amount of kinetic energy contained in the low wavenumbers $k < k_f$ is slightly reduced with respect to the early stage of the evolution.
Summarizing, our results show that, in presence of a force acting on the magnetic field, it is possible to observe the development of a large-scale magnetic condensate in 2D MHD. Nevertheless, this large-scale coherent magnetic field is not able to induce a large-scale "motor effect" on the velocity field.

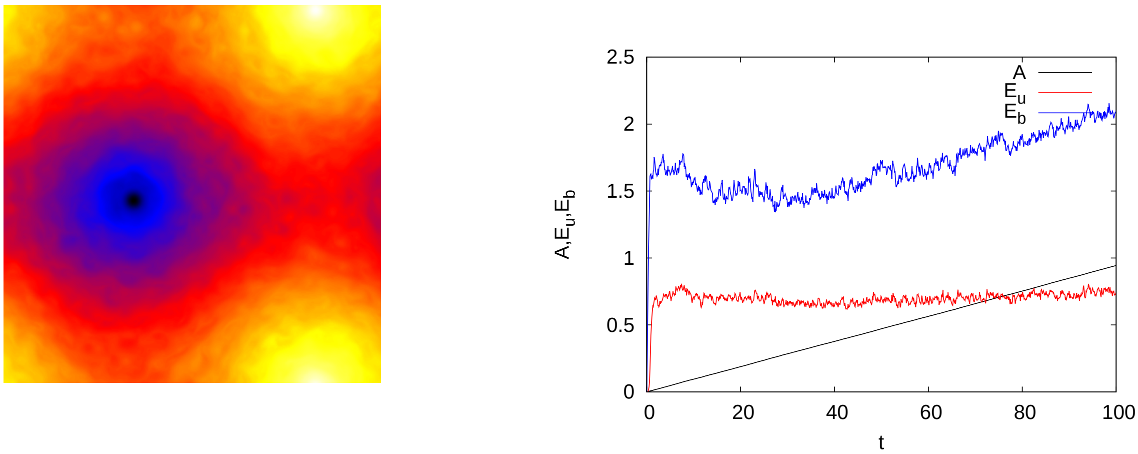

Figure 1. Left: Magnetic potential field a after the formation of the condensed state. Right: Temporal evolution of the the square magnetic potential $A = 1/2\langle a^2 \rangle$ (black), of kinetic energy $E_u = 1/2\langle |\boldsymbol{u}|^2 \rangle$ (red) and magnetic energy $E_b = 1/2\langle |\boldsymbol{b}|^2 \rangle$ (blue)

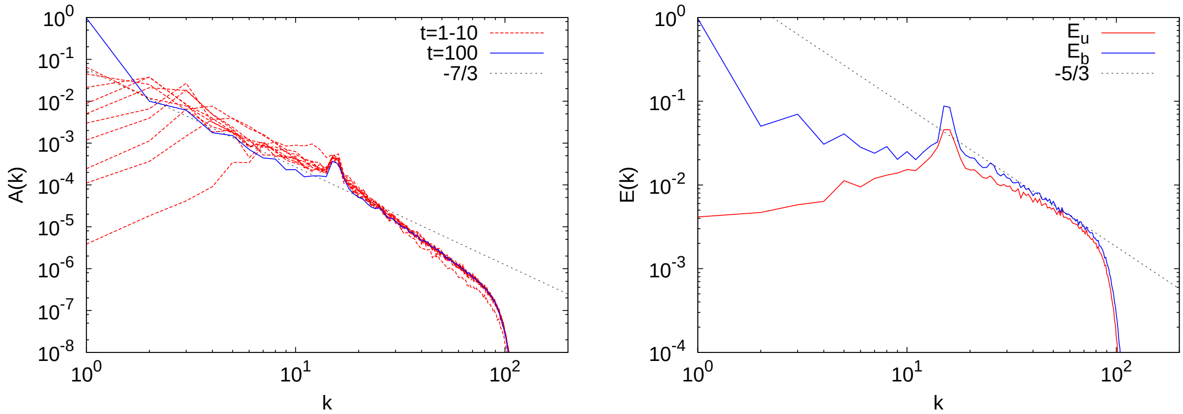

Figure 2. Left panel: Spectra of magnetic potential $A(k)$ at different times: in the early stage, during the development of the inverse cascade ($t \in (1 - 10)$, red), and in the late condensed state ($t = 100$, blue). Right panel: Spectra of kinetic energy E_u (red) and magnetic energy E_b (blue) after the formation of the condensate ($t = 100$).

References

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