NON LOCAL RESONANCES IN WEAK TURBULENCE OF GRAVITY-CAPILLARY WAVES

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Abstract We report a laboratory investigation of weak turbulence of water surface waves in the gravity-capillary crossover. By using time-space resolved profilometry and a bicoherence analysis, we study the 3-wave resonant interactions that are responsible for energy transfer among waves. We show that the energy transfer occurs through non local coupling between capillary and gravity waves.

A large ensemble of nonlinear waves can exchange energy and develop a turbulent state. The statistic properties of such wave turbulence have been described theoretically for weak non linearity in the framework of the Weak Turbulence Theory (WTT). In this theory, only resonant waves are able to exchange significant amounts of energy over long times due to the weak coupling among waves. The predicted phenomenology of the stationary statistical states resembles that of fluid turbulence: energy is injected at large scales and cascades down scale to wavelengths at which dissipation takes over and absorbs energy into heat. A major difference with fluid turbulence is that analytical predictions for the stationary spectra (and other statistical quantities) can be derived for weak wave turbulence [19, 14, 16]. Sea surface waves are among the pioneering physical systems that led to the development of the theory [9]. The theory was subsequently applied to a vast amount of waves (in plasmas [19], solar winds [8], non linear optics [6], quantum superfluid vortices [1], vibrated elastic plates [5],...).

Laboratory experiments largely fail to reproduce the theoretical predictions, in particular for the gravity surface waves. In large or small waves tanks, the spectral exponent of the gravity waves is seen to vary strongly with the forcing intensity and to be close to the WTT predictions at the highest forcing magnitude, at odds with the weak non linearity hypothesis [15, 7]. Recent work on water waves and vibrated plates suggest that wideband dissipation is most likely responsible for the latter observation [11, 13, 4]. Another experiment also suggest that several regimes of wave turbulence of water wave may exist depending on the intensity and frequency of the forcing [2]. In our presentation, we will report a high order statistical analysis that directly probes the non linear interaction among waves. The accessible wave lengths corresponds to capillary waves and to the gravity-capillary crossover.

The experimental setup consist of a rectangular plastic vessel of 70×40 cm^2 filled with water to a rest height $h_0 =$ 5 cm. Surfaces waves are excited by horizontally vibrating the vessel. Waves are measured using the Fourier transform profilometry technique which enables a full space-time characterization of the waves [3]. Similarly to Przadka *et al.* [17] we use anatase titanium dioxide particles (Kronos 1001) that do not alter the pure water surface tension and do not induce additional dissipation at the surface. Thanks to this pigment, it is possible to project a pattern at the very surface of water. When the water surface is deformed, the pattern seen by a camera is changed. The alteration of the pattern can then be inverted to recover the deformation of the surface [12].

We show in fig. 1 the space-time power spectrum $E^v(k,\omega)$ of the velocity field $v = \frac{\partial \eta}{\partial t}$ where $\eta(x,y,t)$ is the altitude of the water surface. We first compute

$$E^v(\mathbf{k},\omega) = \langle |v(\mathbf{k},\omega)|^2 \rangle \tag{1}$$

where $v(\mathbf{k},\omega)$ is the space and time Fourier transform of the velocity. The time Fourier transform is computed by selecting a 16 s time window of the signal. The average $\langle ... \rangle$ is a time average over the time windows. $E^v(\mathbf{k},\omega)$ is then summed over directions of the 2D wavevector $\mathbf{k}$ to provide a 2D picture of $E^v(k,\omega)$. Energy is seen to be concentrated around the dispersion relation of gravity-capillary waves $\omega = \left(gk + \frac{\gamma}{\rho}k^3\right)^{1/2}$ with $k = |\mathbf{k}|$. The isotropy of $E^v(\mathbf{k},\omega)$ is shown in the inset of the Figure 1 at a given frequency of 10 Hz. Energy is convincingly distributed over all directions. The energy concentration around the dispersion relation is due to non linear spectral widening predicted by the WTT framework. Note that no secondary branches of the dispersion relation are seen contrary to what was reported in [10]. Our regime corresponds to the second regime of turbulence reported in [2] at the weakest magnitude of the waves. The wave steepness of our data is indeed small: $\sigma = \left\langle \sqrt{\frac{1}{S}\int_S \|\nabla h(x,y,t)\|^2 dxdy} \right\rangle = 0.025$ thus our wave field is weakly non linear.

To investigate the 3-wave coupling in our experiment, we study third order correlations of the velocity field. From $v(x,y,t)$, we compute the Fourier transform in time over 4 s time windows so that to obtain $v(x,y,\omega)$. Correlations are then computed as

$$C(\omega_1,\omega_2,\omega_3) = \frac{|\langle v^\star(x,y,\omega_1)v(x,y,\omega_2)v(x,y,\omega_3)\rangle|}{[E^v(\omega_1)E^v(\omega_2)E^v(\omega_3)]^{1/2}} \tag{2}$$

where * stands for complex conjugation $E^v(\omega) = \langle\langle |v(x,y,\omega)|^2\rangle\rangle$ is the frequency spectrum. The observed bicoherence $B(\omega_2,\omega_3) = C(\omega_2+\omega_3,\omega_2,\omega_3)$ (correlation on the resonance line $\omega_1 = \omega_2+\omega_3$) is shown in fig. 2. In our presentation

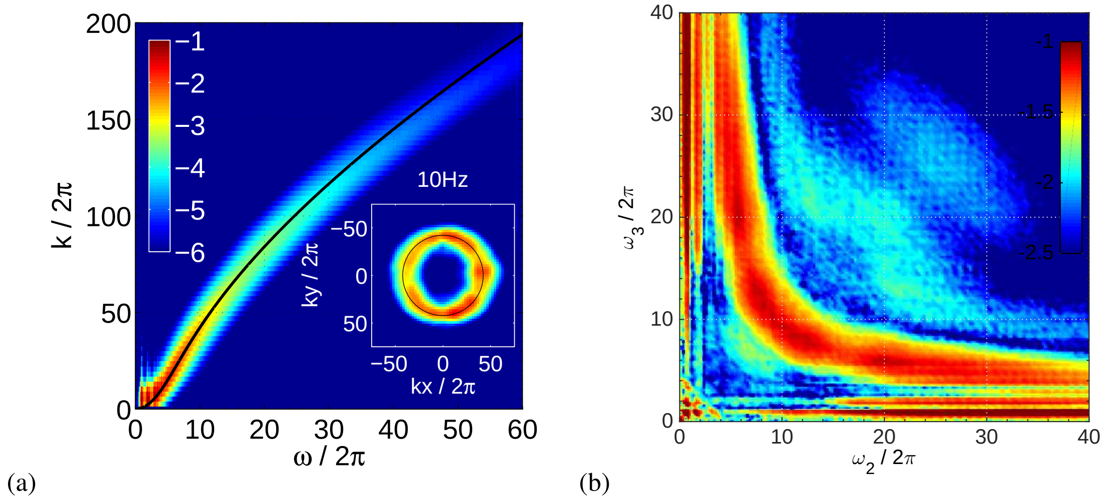

Figure 1. (a) Space-time Fourier spectrum of the velocity field of the waves $E^v(k, \omega)$ (see text for definition). The color scale is $\log_{10}$ coded. The solid black line is the theoretical deep water linear dispersion relation for pure water $\omega^2 = gk + \frac{\gamma}{\rho}k^3$ with $\gamma = 72$ mN/m. Energy is localized on the dispersion relation and can be observed for frequencies up to 60 Hz. The crossover between gravity and capillary waves occurs at $k_c = \sqrt{\rho g/\gamma} = 120\pi$ corresponding to a wavelength of 1.7 cm and a frequency equal to 13 Hz. Inset: $E(\mathbf{k}, \omega)$ at $\omega/2\pi = 10$ Hz. The energy distribution is fairly isotropic. The black circle corresponds to the linear dispersion relation. (b) Bicoherence $B(\omega_2, \omega_3) = C(\omega_2 + \omega_3, \omega_2, \omega_3)$. The color scale corresponds to $\log_{10} B$.

we will detail our understanding of the structure of this bicoherence picture in terms of non local coupling between gravity and capillary waves.

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