HALL EFFECTS ON SCALE-HIERARCHY IN MHD TURBULENCE

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Abstract Direct numerical simulations of MHD turbulence with and without uniform magnetic field are carried out to study the Hall effects on scale-hierarchy in MHD turbulence. It is observed that vortex and current sheets are filamented either by the Kelvin-Helmholtz instability or magnetic reconnection in case of Hall MHD turbulence, while the filamentation is not observed without the Hall term. We show that the filamentation occurs not only for scales smaller than the ion skin depth, which is indicated by the Hall parameter, but also for scales larger than the ion skin depth, affecting turbulence statistics. It is also shown that the Hall effects can be modelled by a Smagorinsky-type model effectively for high wave number regions.

INTRODUCTION

Macroscopic motions of hot plasma in fusion and solar winds are often simulated by solving the single-fluid magneto-hydrodynamic (MHD) equations. However, some additional terms such as the two-fluid (or Hall) term which are truncated in the derivation of the single-fluid MHD equations should be also considered when the small-scale nature of turbulence is focused on, because the additional effects change dynamics of small-scales considerably. For example, a typical value of the ion skin depth in an experiment of the large helical device (LHD), a heliotron-type fusion experiment facility, is 5cm, while a fully nonlinear MHD simulation resolves a scale as short as or shorter than 5mm, and high-wave-number dynamics appear fully turbulent there[1]. Thus we need to study turbulent MHD dynamics with the small-scale effects. The Hall MHD model is among the simplest MHD equations with the small-scale effects. The Hall term is a part of the two-fluid effects in the generalized Ohmic law, and provides many new physics to MHD phenomena[2, 3, 4, 5]. Recently we have shown that the introduction of the Hall term to homogeneous and isotropic turbulence brings about the filamentation of the current sheets and transition of the vortex layer to tubular vortices[5]. Here we study properties of homogeneous Hall MHD turbulence with and without uniform magnetic field by means of direct numerical simulations (DNS) with an intention to highlight influences of the Hall term to the suppression/enhancement of instability which can cause the structure transition as well as the impact of the transition to turbulence statistics and modeling.

STRUCTURE FILAMENTATION

DNSes of homogeneous Hall MHD turbulence with and without uniform magnetic field are carried out by the use of the pseudo-spectral method and the Runge-Kutta-Gill scheme. The number of the grid points are typically $N^3 = 1024^3$. DNSes in which a constant and uniform magnetic field $B_0 = 5$ is imposed are compared to those of $B_0 = 0$. The Hall parameter ϵ_H which control the strength of the Hall term is typically either $\epsilon_H = 0.05, 0.025, 0.0125$, or 0. The magnetic energy spectrum in Hall MHD turbulence ($\epsilon_H \neq 0$) is proportional to $k^{-7/3}$ while it is likely to be proportional to $k^{-5/3}$ in MHD turbulence ($\epsilon_H = 0$).

Firstly, we show that the structure transition from the vortex sheets to tubes and filamentation of the current sheets can occur not only in the scales smaller than the ion skin depth but also in the scales larger than ion skin depth, whether we impose the uniform magnetic field or not. In Fig.1, the isosurfaces of the current density (light grey) and the enstrophy density (blue) in DNSes of MHD turbulence under a uniform magnetic field (a) with and (b) without the Hall term. In Fig.1(a), both the isosurfaces of the enstrophy density and the current density are tubular or filamented, as we have seen in isotropic turbulence[5]. We can show that the structure transitions occur not only in the scales smaller than the ion skin depth but also in the scales larger than it. The filamentated current isosurfaces are relatively inclined to the direction of the uniform magnetic field, while the tubular vortices are inclined relatively orthogonal to the uniform magnetic field. On the other hand, in Fig.1(b), the isosurfaces of the current density and isosurfaces travel almost together, forming thin sheet-like structures. The wavy structures in some sheet structures represent the ocurrence of the Kelvin-Helmholtz instability to the sheets. An observation on the instability shows that filamentation of sheets is not observed as frequently as those in Hall MHD turbulence. It suggests that the structure transition of the current density and the enstrophy density can be closely related with the enhancement of the instability due to the instability by the Hall term as well as the enhancement of magnetic reconnection which are discussed often in the context of Hall effects.

In order to simulate MHD turbulence with the structure transition more quickly, we consider carrying out large eddy simulations of Hall MHD equations. Applicability of a traditional Smagorinsky-type sub-grid-scale (SGS) model to Hall

MHD turbulence is assessed by projecting the Fourier components of DNS data of $k > k_c$ (k_c is an assumed cut-off wave number) to a field based on a SGS model for MHD turbulence developed by Hamba and Tsuchiya[6]. The difference of the original DNS data of $k > k_c$ and the projection of data to the SGS field, the residue of projection, can be a good index to assess the applicability. In Fig.2, the residues of (a) the induction equation (excluding the Hall term) and (b) the Hall term in Hall MHD turbulence for $B_0 = 0$ is shown. It is found that the residue in the induction term decreases smoothly as the function of k_c, while the residue decreases suddenly at $k > 128$ [5]. The observation suggests that the SGS model can be applicable to Hall MHD turbulence for a very high wave number region. The applicability of SGS model to Hal MHD turbulence of $B_0 = 5$ will be studied.

In addition to the results in the above, in the presentation, impact of the instability to the structure transition as well as of the structure transition to turbulence statistics will be presented.

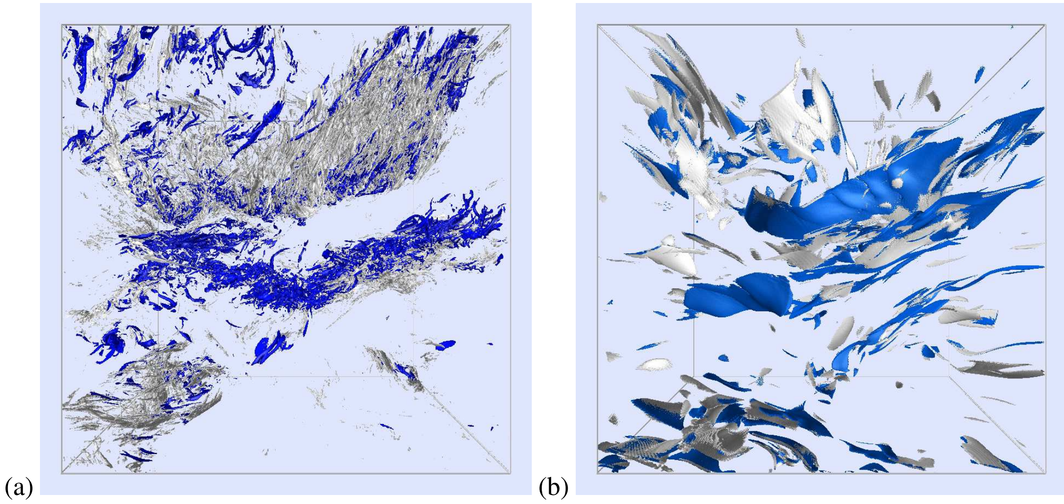

Figure 1. Isosurfaces of the current density (light grey) and the enstrophy density (blue) in DNSes of (a)Hall MHD and (b)MHD turbulence with uniform magnetic field.

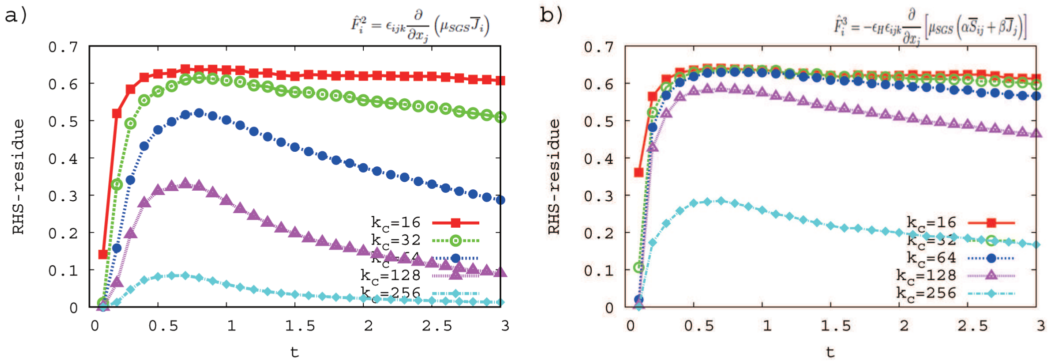

Figure 2. Residues of SGS projection in (a) the induction term but the Hall term and (b) the Hall term. The quantities F^2 and F^3 represents the induction term and the Hall term, respectively, based on the SGS model. The symbols $\overline{S}_{ij}$ and $\overline{J}_i$ are the (i, j) component of the rate-of-strain tensor and the i-th component of the current vector in the grid-scale, respectively, while μ_{SGS} is the eddy-viscosity based on the Smagorinsky model. See [5] for details.

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